Lesson 27 – Solving Rational Equations

Math 2Honors - Santowski

FAST FIVE

- Predict the end behaviour of $f(x)=\frac{x}{x-6}$
- Predict the VA and NVA of $f(x)=\frac{x}{x-6}$
- Predict the end behaviour of $g(x)=\frac{1}{x-4}$
- Predict the VA and NVA of $g(x)=\frac{1}{x-4}$
- HOPEFULLY, as the lesson progresses, you will see the connection between SOLVING EQUATIONS and KNOWING WHAT GRAPHS LOOK LIKE

Lesson Objectives

- Review strategies for solving equations and apply them to rational equations (algebraic, graphic, numeric & checking solutions)
- HOPEFULLY, as the lesson progresses, you will see the connection between SOLVING EQUATIONS and KNOWING WHAT GRAPHS LOOK LIKE
- Solve equations in the context of applications & modeling

(A) Review

- We can solve ANY given equation in a variety of ways:
- (a) Numeric
- (b) Graphic
- (c) Algebraic

(B) Solving Numerically – Example 1

- Solve the following equation, given the following table of values:

$$\frac{x}{x-6}=\frac{1}{x-4}$$

- The table of values shows that the solution is ……??
- Since the table of values only lies between $\{x \in I \mid -4 < x < 5\}$, are there OTHER solutions:
- (i) beyond either x = -4 or x = 5?
- (ii) are there non-integral solutions??

x	f(x)	g(x)
-4	0.4	-0.125
-3	0.333…	-0.1429…
-2	0.25	-0.16666..
-1	0.1429…	-0.2
0	0	-0.25
1	-0.2	-0.3333..
2	-0.5	-0.5
3	-1	-1
4	-2	Undefined
5	-5	1

(B) Solving Numerically – Example 1

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- (ii) are there non-integral solutions??

x	f(x)	g(x)
5	-5	1
6	Error	0.5
7	7	0.3333…
8	4	0.25
9	3	0.2
10	2.5	0.1666..
11	2.2	0.1428…
12	2	0.125
13	1.875…	0.1111…
14	1.75	0.1

(B) Solving Numerically – Example 1 Solution

- Solve $\frac{x}{x-6}=\frac{1}{x-4}$
- The table of values shows that the solution is x = 2 and x = 3 since the 2 functions have the same function values at these x values
- (i) beyond x = -4 &5 (No since end behaviours ($\pm\infty$ are different 1 and 0)
- (ii) are there non-integral solutions?? (No, since one function is always above the other in each integral interval)

x	f(x)	g(x)
-4	0.4	-0.125
-3	0.333…	-0.1429…
-2	0.25	-0.16666..
-1	0.1429…	-0.2
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1	-0.2	-0.3333..
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(C) Solving Graphically

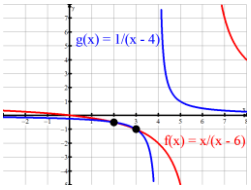
- Solve by graphing: $\frac{x}{x-6}=\frac{1}{x-4}$
- When we solve by graphing, WHAT do we look for???

(C) Solving Graphically

- Solve by graphing:

$$\frac{x}{x-6}=\frac{1}{x-4}$$

- When we solve by graphing, WHAT do we look for???
- We are looking for the x co-ordinate of the intersection point



(C) Solving Graphically

- Solve by graphing:

$$\begin{cases} f(x)=\frac{x}{x-6} \\ g(x)=\frac{1}{x-4} \end{cases}$$

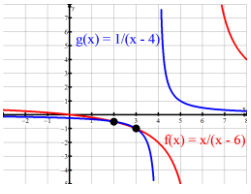
- When we solve by graphing, WHAT do we look for???

(C) Solving Graphically

- Solve by graphing:

$$\begin{cases} f(x)=\frac{x}{x-6} \\ g(x)=\frac{1}{x-4} \end{cases}$$

- When we solve by graphing, WHAT do we look for???
- We are looking for the x and y co-ordinates of the intersection point **(2,-0.5) and (3,-1)**



(C) Solving Graphically

- Solve by graphing: $\frac{x}{x-6}=\frac{1}{x-4}$
- But we will set up an ALTERNATIVE graphing approach ➔ what's different NOW???

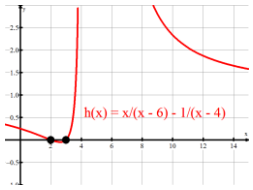
$$\frac{x}{x-6}-\frac{1}{x-4}=0$$

- When we solve by graphing with this alternative method, WHAT do we look for???

(C) Solving Graphically

- Solve by graphing:

$$\frac{x}{x-6}-\frac{1}{x-4}=0$$



- When we solve by graphing, WHAT do we look for???
- We are looking for the x intercepts or roots or zeroes of the rational function

$$\frac{x}{x-6}-\frac{1}{x-4}=h(x)$$

(D) Solving Algebraically

- Solve for x algebraically $\frac{x}{x-6}=\frac{1}{x-4}$
- So now we must firstly consider RESTRICTIONS upon the independent variable, x ➔ WHY???

(D) Solving Algebraically

- Solve for x algebraically $\frac{x}{x-6}=\frac{1}{x-4}$

$$\frac{x}{x-6}=\frac{1}{x-4}$$
$$\left(\frac{x-4}{x-4}\right)\left(\frac{x}{x-6}\right)=\left(\frac{1}{x-4}\right)\left(\frac{x-6}{x-6}\right);x\neq 4,6$$
$$x(x-4)=1(x-6)$$
$$x^2-4x-x+6=0$$
$$x^2-5x+6=0$$
$$(x-3)(x-2)=0$$
$$\therefore x=2,3 \quad \text{but} \quad x\neq 4,6$$

(D) Solving Algebraically

- in our algebraic solution, we algebraically created an EQUIVALENT system

$$\frac{x}{x-6}=\frac{1}{x-4}$$
became
$$x^2-5x+6=0$$

- EXPLAIN what we mean by an EQUIVALENT system????

(D) Solving Algebraically

- in our algebraic solution, we algebraically created an EQUIVALENT system

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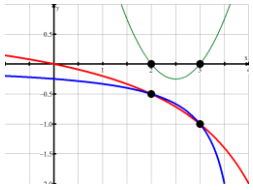
- EXPLAIN what we mean by EQUIVALENT systems????
- Now explain WHY we need to check our algebraic solutions?



(B) Solving Numerically – Example 2

- Solve the following equation, given the following table of values:

$$\frac{x}{x-6}=\frac{1}{x-4.25}$$

- The table of values shows that the solution is ……??
- Since the table of values only lies between {x E I| -4 < x < 5}, are there OTHER solutions:
- (i) beyond either x = -4 or x = 5?
- (ii) are there non-integral solutions??

x	f(x)	g(x)
-4	0.4	-0.1212…
-3	0.333…	-0.1379…
-2	0.25	-0.16
-1	0.1429…	-0.1905…
0	0	-0.2353…
1	-0.2	-0.3077…
2	-0.5	-0.4444…
3	-1	-0.8
4	-2	-4
5	-5	1.3333…

(E) Practice

- Now that we have reviewed 3 different solving strategies, work through the following examples using the given strategies:
- (i) algebraic and graphic $\frac{x}{x-3}+\frac{2x}{x+3}=\frac{18}{x^2-9}$
- (ii) algebraic and numeric $\frac{x}{x-2}+\frac{x}{x-3}=\frac{3}{x^2-5x+6}$

(E) Practice

- Now that we have reviewed 3 different solving strategies, work through the following examples using the given strategies:
- (i) algebraic and graphic $\frac{x+2}{2x-3}+\frac{x-2}{2x+3}=\frac{21}{4x^2-9}$
- (ii) algebraic and numeric $\frac{3x}{x-1}+\frac{2x}{x-6}=\frac{5x^2-15x+20}{x^2-7x+6}$

(F) Applications of Rational Equations

- Ex 1. Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and 39 km back. Find the speed of the current to two decimal places.
- Ex 2. Jaime bought a case of concert T-shirts for $450. She kept two for herself and sold the rest for $560, making a profit of $10 on each shirt. How many shirts were in the case?

(F) Applications of Rational Equations

- Mr S. and Mr. E set off at the same time on a 30K walk. Mr. E. runs 1.4 km/h faster than Mr. S. Mr. E. finishes 2 hrs ahead of Mr. S. in spite of a 20 minute rest in the race. How fast was each one running and how long did it take each person to run the race?

(F) Applications of Rational Equations

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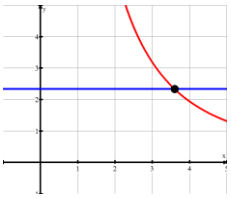
- Equation is

$$\frac{30}{v_s}-\frac{30}{v_s+1.4}=2\frac{1}{3}$$



- Solution is v_s = 3.6 km/h and it took me 8 hours & 20 min

(G) Homework

- Textbook, p. 517 # 9-27 odd, 47-50, 54, 56-57