

Lesson 26 – Operations with Rational Expressions

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(A) Multiplying and Dividing With Fractions

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- recall how to divide and multiply rational numbers - fractions
- ex. $(6/5) \times (25/42)$ - multiply the numerator and then multiply the denominator $\rightarrow (150/210) = 5/7$
- \rightarrow OR look to cancel COMMON FACTORS between the numerator and denominators prior to multiplying
- $\rightarrow (6/5) \times (5 \times 5)/(6 \times 7) = (1/1) \times (5/7) = 5/7$
- ex. $(5/12) \div (15/64)$ - multiply by the reciprocal $(5/12) \times (64/15) \rightarrow (5/4 \times 3) \div (4 \times 16/3 \times 5) \rightarrow (16/3 \times 3) \rightarrow 16/9$

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(A) Multiplying and Dividing With Fractions – Example 1

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- the strategy that will be used - we must try to factor the expressions in the numerator and denominator
- Example: Simplify $\frac{7ab}{3c} \times \frac{21ac^2}{5a^2}$
- But are there restrictions in this case? Why?

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(A) Multiplying and Dividing With Fractions – Solution for Example 1

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- Example: Simplify $\frac{7ab}{3c} \times \frac{21ac^2}{5a^2}$
- $$\begin{aligned} & \frac{7ab}{3c} \times \frac{21ac^2}{5a^2} \\ &= \frac{7 \times a \times b}{3 \times c} \times \frac{7 \times 3 \times a \times c \times c}{5 \times a \times a}; a \neq 0, c \neq 0 \\ &= \frac{7b}{1} \times \frac{7c}{5} \\ &= \frac{49bc}{5}; a \neq 0, c \neq 0 \end{aligned}$$

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(A) Multiplying and Dividing With Rational Expressions – Example 2

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- Simplify and state restrictions for

$$\frac{y+2}{3a} \div \frac{2y+4}{9a}$$

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(A) Multiplying and Dividing With Rational Expressions – Solution for Example 2

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- Simplify $\frac{y+2}{3a} \div \frac{2y+4}{9a}$
- $$\begin{aligned} & \frac{y+2}{3a} \div \frac{2y+4}{9a} \\ &= \frac{1(y+2)}{3a} \times \frac{3(3a)}{2(y+2)}; a \neq 0; y \neq -2 \\ &= \frac{3}{2}; a \neq 0; y \neq -2 \end{aligned}$$

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(A) Multiplying and Dividing With Rational Expressions – Example 3

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- Simplify and state restrictions for:

$$\frac{a^2 - 2a - 3}{a^2 + 6a + 5} \times \frac{a^2 + 7a + 10}{a^2 - a - 6}$$

- Graphically, verify your answer

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(A) Multiplying and Dividing With Rational Expressions – Solution for Example 3

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- Simplify $\frac{a^2 - 2a - 3}{a^2 + 6a + 5} \times \frac{a^2 + 7a + 10}{a^2 - a - 6}$

$$\begin{aligned} & \frac{a^2 - 2a - 3}{a^2 + 6a + 5} \times \frac{a^2 + 7a + 10}{a^2 - a - 6} \\ &= \frac{(a-3)(a+1)}{(a+5)(a+1)} \times \frac{(a+5)(a+2)}{(a-3)(a+2)}; a \neq -5, -1, 3, -2 \end{aligned}$$

$$= \frac{1}{1}; a \neq -5, -1, 3, -2$$

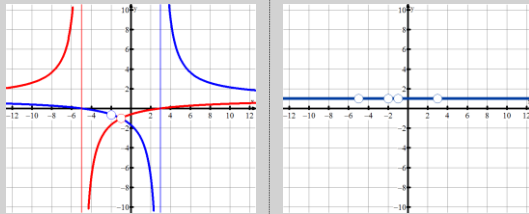
$$= 1; a \neq -5, -1, 3, -2$$

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(A) Multiplying and Dividing With Rational Expressions – Solution for Example 3

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(A) Multiplying and Dividing With Rational Expressions – Example 4

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- Simplify and state restrictions

$$\frac{2d^2 - 3d - 2}{3d^2 - 8d + 4} \div \frac{2d^2 + 3d + 1}{3d^2 - 2d}$$

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(A) Multiplying and Dividing With Rational Expressions – Solution for Example 4

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- Simplify $\frac{2d^2 - 3d - 2}{3d^2 - 8d + 4} \div \frac{2d^2 + 3d + 1}{3d^2 - 2d}$

$$\begin{aligned} & \frac{2d^2 - 3d - 2}{3d^2 - 8d + 4} \div \frac{2d^2 + 3d + 1}{3d^2 - 2d} \\ &= \frac{(2d+1)(d-2)}{(3d-2)(d-2)} \times \frac{d(3d-2)}{(d+1)(2d+1)} \\ &= \frac{d}{d+1}; d \neq -1, 2, \frac{2}{3}, -\frac{1}{2}, 0 \end{aligned}$$

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(A) Multiplying and Dividing With Rational Expressions – Example 5

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- We introduce an idea called a complex fraction:
- Consider the 2 expressions

$$\frac{\frac{x^2}{x^2-1}}{\frac{x}{x^2+2x-3}} \quad \text{and} \quad \frac{x^2}{x^2-1} \div \frac{x}{x^2+2x-3}$$

- Is there any difference?
- How do you then simplify a complex fraction?

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(A) Multiplying and Dividing With Rational Expressions – Solution to Example 5

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- Simplify the complex fraction

$$\begin{aligned} & \frac{\frac{x^2}{x^2-1}}{\frac{x}{x^2+2x-3}} \\ &= \frac{x^2}{x^2-1} \div \frac{x}{x^2+2x-3} \\ &= \frac{x^2}{(x-1)(x+1)} \times \frac{(x+3)(x-1)}{x}; x \neq -3, -1, 0, 1 \\ &= \frac{x(x+3)}{(x+1)} \\ &= \frac{x^2+3x}{x+1}; x \neq -3, -1, 0, 1 \end{aligned}$$

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(B) Adding & Subtracting With Fractions

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- To add and subtract fractions:
 - find a common denominator
 - convert fractions to equivalent fractions & operate
- $2/5 + 3/7 = 2(7) / 5(7) + 3(5) / 7(5)$
- $= 14/35 + 15/35 = 29/35$
- $3/13 - 5/4 = 3(4) / 13(4) - 5(13) / 4(13)$
- $= 12/52 - 65/52 = -53/52$

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(B) Adding & Subtracting With Rational Expressions – Example 1

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- Simplify and state restrictions $\frac{3}{a} + \frac{4}{b}$

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(B) Adding & Subtracting With Rational Expressions – Solution for Example 1

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- Simplify and state restrictions $\frac{3}{a} + \frac{4}{b}$
- $$\begin{aligned} & \frac{3}{a} + \frac{4}{b} \\ &= \frac{3(b)}{a(b)} + \frac{4(a)}{b(a)} \\ &= \frac{3b+4a}{ab}; a \neq 0, b \neq 0 \end{aligned}$$

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(B) Adding & Subtracting With Rational Expressions – Example 2

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- Simplify and state restrictions $\frac{4}{1-w} - \frac{3}{w+1}$
- Graphically verify your work

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(B) Adding & Subtracting With Rational Expressions – Solution for Example 2

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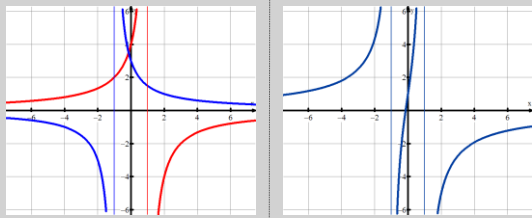
- Simplify and state restrictions $\frac{4}{1-w} - \frac{3}{w+1}$
- $$\begin{aligned} & \frac{4}{1-w} - \frac{3}{w+1} \\ &= \frac{4(w+1)}{(1-w)(w+1)} - \frac{3(1-w)}{(w+1)(1-w)} \\ &= \frac{4(w+1) - 3(1-w)}{(1-w)(w+1)} \\ &= \frac{7w+1}{(1-w)(w+1)}; w \neq \pm 1 \end{aligned}$$

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(B) Adding & Subtracting With Rational Expressions – Solution for Example 2

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(B) Adding & Subtracting With Rational Expressions – Example 3

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- Simplify and state restrictions for

$$\frac{1}{a+b} + \frac{1}{a-b} - \frac{2b}{a^2-b^2}$$

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(B) Adding & Subtracting With Rational Expressions – Solution for Example 3

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- Simplify and state restrictions for $\frac{1}{a+b} + \frac{1}{a-b} - \frac{2b}{a^2-b^2}$

$$\begin{aligned} & \frac{1}{a+b} + \frac{1}{a-b} - \frac{2b}{a^2-b^2} \\ &= \frac{1(a-b)}{(a+b)(a-b)} + \frac{1(a+b)}{(a-b)(a+b)} - \frac{2b}{(a-b)(a+b)} \\ &= \frac{1(a-b) + 1(a+b) - 2b}{(a+b)(a-b)} \\ &= \frac{2a-2b}{(a+b)(a-b)} \\ &= \frac{2(a-b)}{(a+b)(a-b)} = \frac{2}{a+b}; a \neq \pm b \end{aligned}$$

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(B) Adding & Subtracting With Rational Expressions – Example 4

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- Simplify and state restrictions

$$\frac{1}{x^2-6x+9} - \frac{1}{x^3-9x}$$

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(B) Adding & Subtracting With Rational Expressions – Solution for Example 4

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- Simplify and state restrictions $\frac{1}{x^2-6x+9} - \frac{1}{x^3-9x}$

$$\begin{aligned} & \frac{1}{x^2-6x+9} - \frac{1}{x^3-9x} \\ &= \frac{1}{(x-3)(x-3)} - \frac{1}{x(x^2-9)} \\ &= \frac{1}{(x-3)(x-3)} - \frac{1}{x(x-3)(x+3)} \\ &= \frac{x(x+3)}{x(x-3)(x-3)(x+3)} - \frac{(x-3)}{x(x-3)(x-3)(x+3)} \\ &= \frac{x^2+3x-x+3}{x(x+3)(x-3)^2} \\ &= \frac{x^2+2x+3}{x(x+3)(x-3)^2}; x \neq 0, \pm 3 \end{aligned}$$

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(B) Adding & Subtracting With Rational Expressions – Example 5

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- Simplify and state restrictions

$$\frac{x+1}{x^2+2x-3} - \frac{x+2}{x^2+4x-5}$$

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(B) Adding & Subtracting With Rational Expressions – Solutions for Example 5

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- Simplify and state restrictions $\frac{x+1}{x^2+2x-3} - \frac{x+2}{x^2+4x-5}$

$$\begin{aligned} & \frac{x+1}{x^2+2x-3} - \frac{x+2}{x^2+4x-5} \\ &= \frac{x+1}{(x+3)(x-1)} - \frac{x+2}{(x+5)(x-1)} \\ &= \frac{(x+1)(x+5) - (x+2)(x+3)}{(x+3)(x-1)(x+5)} \\ &= \frac{(x^2+6x+5) - (x^2+5x+6)}{(x+3)(x-1)(x+5)} \\ &= \frac{x-1}{(x+3)(x-1)(x+5)} \\ &= \frac{1}{(x+3)(x+5)}; x \neq -5, -3, 1 \end{aligned}$$

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(C) Adding & Subtracting With Rational Expressions – Example 6

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- Mr S. and Mr. E set off at the same time on a 30K run. Mr. E. runs 1.4 km/h faster than Mr. S. Mr. E. finishes 2 hrs ahead of Mr. S. in spite of a 20 minute rest in the race. How fast was each one running and how long did it take each person to run the race?

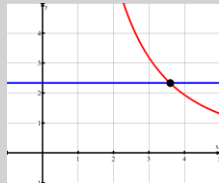
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(C) Adding & Subtracting With Rational Expressions – Solution to Example 6

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- Mr S. and Mr. E set off at the same time on a 30K walk. Mr. E. runs 1.4 km/h faster than Mr. S. Mr. E. finishes 2 hrs ahead of Mr. S. in spite of a 20 minute rest in the race. How fast was each one running and how long did it take each person to run the race?



- Equation is $\frac{30}{v_s} - \frac{30}{v_s + 1.4} = 2\frac{1}{3}$

- Solution is $v_s = 3.6$ km/h and it took me 8 hours & 20 min

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(D) Internet Links

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- [Adding and Subtracting Rational Expressions from West Texas A&M](#)
- [Adding & Subtracting Rationals Lesson - I from Purple Math](#)
- [College Algebra Tutorial on Multiplying and Dividing Rational Expressions from West Texas A&M](#)
- [Multiplying & Dividing Rationals Lesson - I from Purple Math](#)

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(E) Homework

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- Textbook, S8.4, p502, Q13,17-25odds, 31,33,44,46
- Textbook, S8.5, p509, Q15,19,21,29,36,37,43,45,48-50

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