Lesson 25 – Graphs of Rational Functions

Math 2 Honors - Santowski

Lesson Objectives

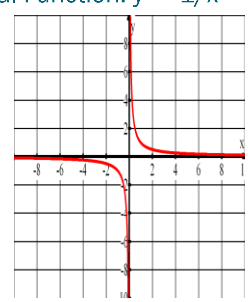
- The next class of functions that we will investigate is the rational functions. We will explore the following ideas:
- The basic rational function, y = 1/x
- Definition of rational function.
- Apply our analysis of the basic rational function to rational functions

(A) The Basic Rational Function: y = 1/x

- Sketch the graph of f(x) = 1/x and list a table of values
- Analyze this parent function for: domain, range, intercepts, intervals of increase/decrease, extrema, end behaviour
- By looking at the **equation** f(x) =1/x, determine the domain.
- By looking at the **graph** of f(x) =1/x, determine the range.

(A) The Basic Rational Function: y = 1/x

x	y
-5.00000	-0.20000
-4.00000	-0.25000
-3.00000	-0.33333
-2.00000	-0.50000
-1.00000	-1.00000
0.00000	undefined
1.00000	1.00000
2.00000	0.50000
3.00000	0.33333
4.00000	0.25000
5.00000	0.20000



(A) The Basic Rational Function: y = 1/x

- The basic features of this basic rational function are:
- (a) domain ➔
- (b) range ➔
- (c) intercepts ➔
- (d) intervals of increase/decrease ➔
- (e) extrema ➔
- (f) end behaviour ➔
- (g) NEW IDEA ➔ DISCONTINUITIES

(A) The Basic Rational Function: y = 1/x

- The basic (untransformed) rational function is f(x) =1/x.
- Complete the following table of values for y if x = {0.0001,0.001,0.01,0.1, 0,1,10,100,1000,10000}.

x	10000	1000	100	10	1	0.1	0.01	0.001	0.0001
y									

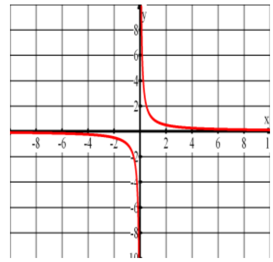
(A) The Basic Rational Function: y = 1/x

- The statement $x \to \infty$ is read "x approaches infinity".
- To approach a value means to get close to the value but not necessarily equal to the value.
- From the table of positive x –values, what value does y approach as $x \to \infty$?
- From the table of negative x –values, what value does y approach as $x \to -\infty$?

- The statement $x \to 0$ is read "x approaches zero".
- From the table of positive x –values, what value does y approach as $x \to 0$?

(A) The Basic Rational Function: y = 1/x

- Use the graph of f(x) =1/x to evaluate the following function behaviours:



(a) $as\ x \to +\infty, y \to ????$

(b) $as\ x \to \infty, y \to ????$

(c) $as\ x \to 0^+, y \to ????$

(d) $as\ x \to 0^-, y \to ????$

(e) $as\ x \to 2, y \to ????$

(f) $as\ x \to \frac{4}{3}, y \to ????$

(B) Asymptotic Behaviour:

- Asymptotic Behaviour: The behaviour of the y – value of graphs to reach bounded values (limits) or to grow unboundedly (to approach infinity) is called "Asymptotic Behaviour".
- Asymptotes are boundary lines (or curves) that act as limiters or attractors of the shape of the graph.

- Write the equation(s) of the asymptote(s) of y = 1/x

- The asymptotes of y = 1/x are referred to as vertical and non-vertical ➔ explain why

(C) Rational Functions

- Just as we saw how the Division Algorithm for integers applies to polynomials or functions, the definition of rational numbers can be extended to functions.

- A rational function is any function of the form r(x) = n(x)/d (x), where n(x) and d(x) represent numerator and denominator polynomials.

- Rational functions then are RATIOS of polynomials.

(D) Exercises

- Analyze the following rational functions:

(a) $y=\frac{1}{x}$ (b) $y=\frac{1}{x-3}$ (c) $y=-\frac{4}{x-3}$

(d) $y=\frac{x+2}{x^2-4}$ (e) $y=\frac{x-2}{x+1}$ (f) $y=\frac{3}{5-2x}$

(g) $y=\frac{2x+2}{x^2-3x-4}$ (h) $y=\frac{x+3}{x^2-3x-4}$ (i) $y=-\frac{1-x^2}{x}$

(j) $y=\frac{1}{x^2}$ (k) $y=\frac{6}{4-2x^2}$ (l) $y=\frac{x^2+1}{x^2-x-2}$

(E) Examples

- Analyze the following rational functions:

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(d) $y=\frac{x+2}{x^2-4}$ (e) $y=\frac{x-2}{x+1}$ (f) $y=\frac{3}{5-2x}$

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