Lesson 22 – Solving Polynomial Equations

Math 2 Honors - Santowski

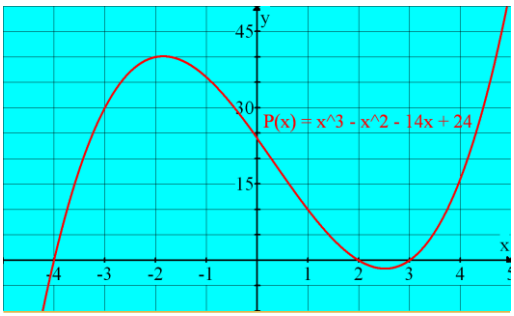
Lesson Objectives

- Mastery of the factoring of polynomials using the algebraic processes of long & synthetic division
- Mastery of the algebraic processes of solving polynomial equations by factoring (Factor Theorem)
- Investigate how equations can be factored and solved graphically, numerically, and by technology
- Reinforce the understanding of the connection between factors and roots

Fast Five

- Solve $x^3 – x^2 – 14x + 24 = 0$, knowing that x = -4 is one of the solutions.

Fast Five – Graph of $P(x) = x^3 – x^2 - 14x + 24$

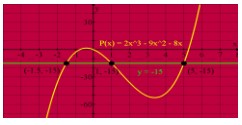


(A) Examples

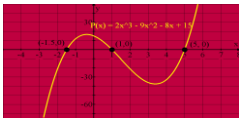
- ex.1 Solve $2x^3 – 9x^2 - 8x = -15$ and then show on a GDC
- ie. Solve the system $\begin{cases} y = 2x^3 - 9x^2 - 8x \\ y = -15 \end{cases}$
- ex 2. Solve $2x^3 + 14x - 20 = 9x^2 - 5$ and then show on a GDC
- ie. Solve the system $\begin{cases} y = 2x^3 + 14x - 20 \\ y = 9x^2 - 5 \end{cases}$

(A) Examples - Solutions

- Solve $2x^3 – 9x^2 - 8x = -15$ and then show on a GDC
- Now graph both
- $g(x) = 2x^3 – 9x^2 - 8x$ and then
- $h(x) = -15$ and find intersection

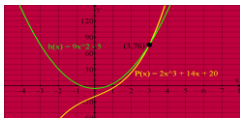


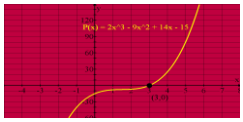
- Then graph:
- $f(x) = 2x^3 – 9x^2 - 8x + 15$



(A) Examples - Solutions

- Solve $2x^3 + 14x - 20 = 9x^2 - 5$ and then show on a GDC
- Explain that different solution sets are possible depending on the number set being used (real or complex)



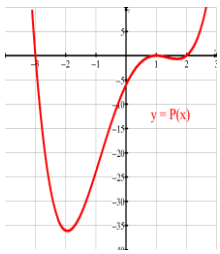


(A) Examples

- Solve $x^4 - x^3 - 7x^2 + 13x - 6 = 0$ then graph using roots, points, end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease (HINT for domain of solution ➔ use RRT}

(A) Examples - Solutions

- Solve $x^4 - x^3 - 7x^2 + 13x - 6 = 0$
- Then graph using roots, points, end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease.
- $P(x) = (x - 1)^2(x + 3)(x - 2)$

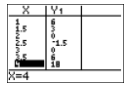


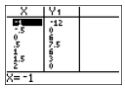
(B) Solving & Factoring on the TI-84

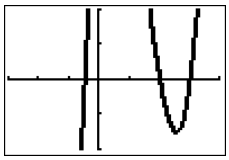
- Factor & Solve the following:
- $0 = 2x^3 - 9x^2 + 7x + 6$
- $3x^3 - 7x^2 + 8x - 2 = 0$
- $x^4 - x^3 - 7x^2 + 13x - 6 = 0$

(B) Solving & Factoring on the TI-84

- Factor & Solve the following:
- $0 = 2x^3 - 9x^2 + 7x + 6$ ➔ roots at x = -0.5,2,3 ➔ would imply factors of (x – 2), (x – 3) and (x + ½) ➔ P(x) = 2(x + ½)(x – 2)(x – 3)
- So when factored P(x) = (2x + 1)(x – 2)(x – 3)

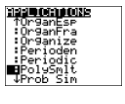


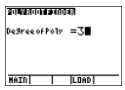


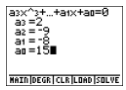


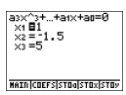
(C) Solving & Factoring on the TI-84

- Solve $2x^3 - 9x^2 - 8x = -15$ turn it into a "root" question ➔ i.e Solve P(x) = 0 ➔ Solve $0 = 2x^3 - 9x^2 - 8x + 15$









(D) Examples - Applications

- You have a sheet of paper 30 cm long by 20 cm wide. You cut out the 4 corners as squares and then fold the remaining four sides to make an open top box.
  - (a) Find the equation that represents the formula for the volume of the box.
  - (b) Find the volume if the squares cut out were each 2 cm by 2 cm.
  - (c) What are the dimensions of the squares that need to be removed if the volume is to be 1008 cm^3?

(D) Examples - Applications

- The volume of a rectangular-based prism is given by the formula $V(x) = -8x + x^3 – 5x^2 + 12$
  - (i) Express the height, width, depth of the prism in terms of *x*
  - (ii) State any restrictions for *x*. Justify your choice
  - (iii) what would be the dimensions on a box having a volume of 650 cubic units?
  - (iv) now use graphing technology to generate a reasonable graph for V(x). Justify your window/view settings

(E) Examples - Applications

- The equation $p(m) = 6m^5 – 15m^4 – 10m^3 + 30m^2 + 10$ relates the production level, p, in thousands of units as a function of the number of months of labour since October, m.
- Use graphing technology to graph the function and determine the following:
  - maximums and minimums. Interpret in context
  - Intervals of increase and decrease. Interpret
  - Explain why it might be realistic to restrict the domain. Explain and justify a domain restriction
  - Would $0 \le m \le 3$ be a realistic domain restriction?
- Find when the production level is 15,500 units (try this one algebraically as well)

(F) Internet Links - Solving Polynomials

- Finding Zeroes of Polynomials from WTAMU
- Finding Zeroes of Polynomials Tutorial #2 from WTAMU
- Solving Polynomials from Purple Math

(G) Homework

- Textbook S7.4
- P453, Q15,21,27,29,33,35,39,43,49,51,56,57,58,