
Lesson 21 – Roots of Polynomial Functions

Math 2 Honors - Santowski

Lesson Objectives

- Reinforce the understanding of the connection between factors and roots
- Mastery of the factoring of polynomials using the algebraic processes of long & synthetic division & various theorems like RRT, RT & FT
- Introduce the term “multiplicity of roots” and illustrate its graphic significance
- Solve polynomial equations for x being an element of the set of complex numbers
- State the Fundamental Theorem of Algebra

(A) Multiplicity of Roots

- Factor the following polynomials:
- $P(x) = x^2 - 2x - 15$
- $P(x) = x^2 - 14x + 49$
- $P(x) = x^3 + 3x^2 + 3x + 1$
- Now solve each polynomial equation, $P(x) = 0$

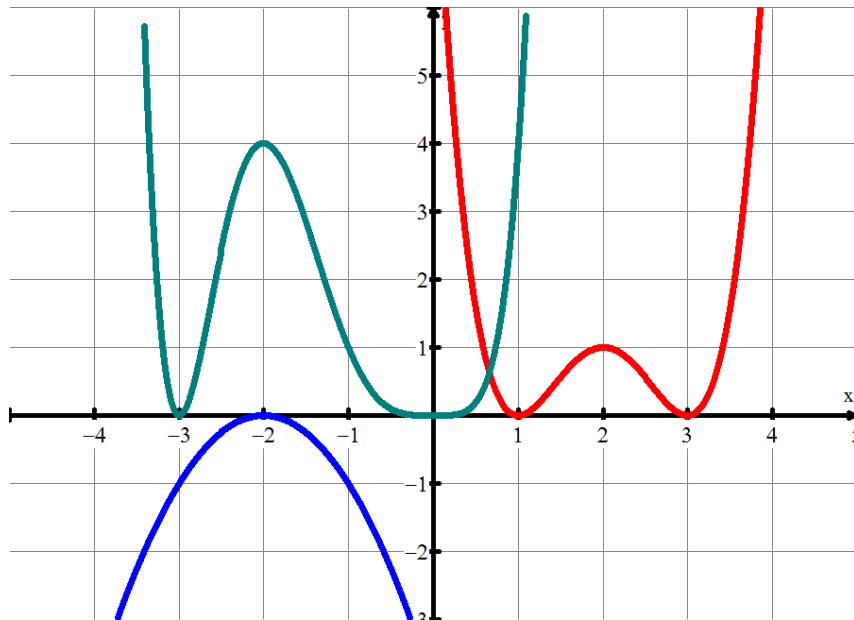
- Solve $0 = 5(x + 1)^2(x - 2)^3$
- Solve $0 = x^4(x - 3)^2(x + 5)$
- Solve $0 = (x + 1)^3(x - 1)^2(x - 5)(x + 4)$

(A) Multiplicity of Roots

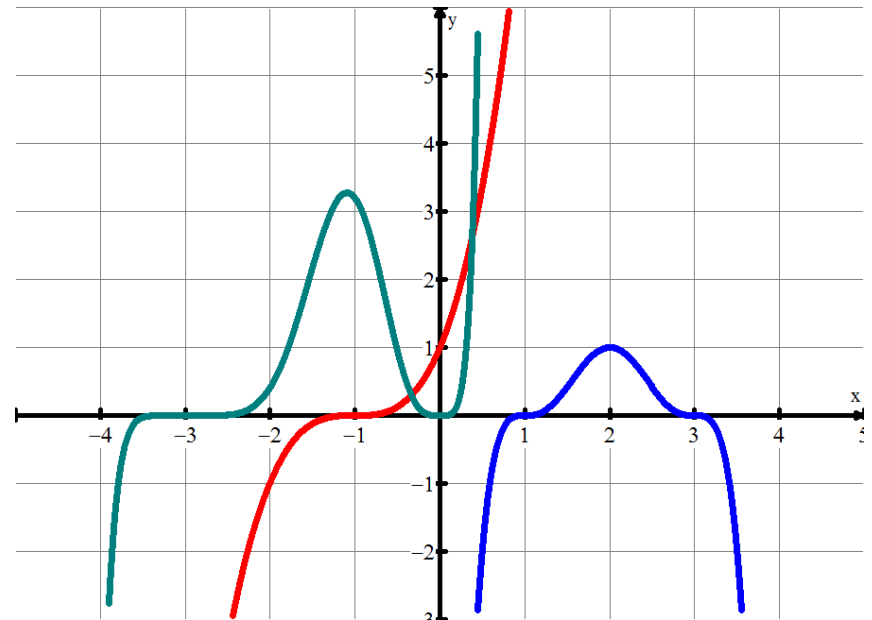
- If r is a zero of a polynomial and the exponent on the factor that produced the root is k , $(x - r)^k$, then we say that r has **multiplicity** of k . Zeroes with a multiplicity of 1 are often called **simple** zeroes.
- For example, the polynomial $x^2 - 14x + 49$ will have one zero, $x = 7$, and its multiplicity is 2. In some way we can think of this zero as occurring twice in the list of all zeroes since we could write the polynomial as, $(x - 7)^2 = (x - 7)(x - 7)$
- Written this way the term $(x - 7)$ shows up twice and each term gives the same zero, $x = 7$.
- Saying that the multiplicity of a zero is k is just a shorthand to acknowledge that the zero will occur k times in the list of all zeroes.

(A) Multiplicity \rightarrow Graphic Connection

Even Multiplicity



Odd Multiplicity



(B) Solving if $x \in \mathbf{C}$

- Let's expand our number set from real numbers to complex numbers
- Factor and solve $3 - 2x^2 - x^4 = 0$ if $x \in \mathbf{C}$
- Factor and solve $3x^3 - 7x^2 + 8x - 2 = 0$ if $x \in \mathbf{C}$
- Factor and solve $2x^3 + 14x - 20 = 9x^2 - 5$ if $x \in \mathbf{C}$
- Now write each polynomial as a product of its factors
- Explain the graphic significance of your solutions for x

(B) Solving if $x \in \mathbf{C}$ – Solution to Ex 1

- Factor and solve $3 - 2x^2 - x^4 = 0$ if $x \in \mathbf{C}$ and then write each polynomial as a product of its factors
- Solutions are $x = \pm 1$ and $x = \pm i\sqrt{3}$
- So rewriting the polynomial in factored form (over the reals) is $P(x) = -(x^2 + 3)(x - 1)(x + 1)$ and over the complex numbers:
$$P(x) = -(x - 1)(x + 1)(x - i\sqrt{3})(x + i\sqrt{3})$$

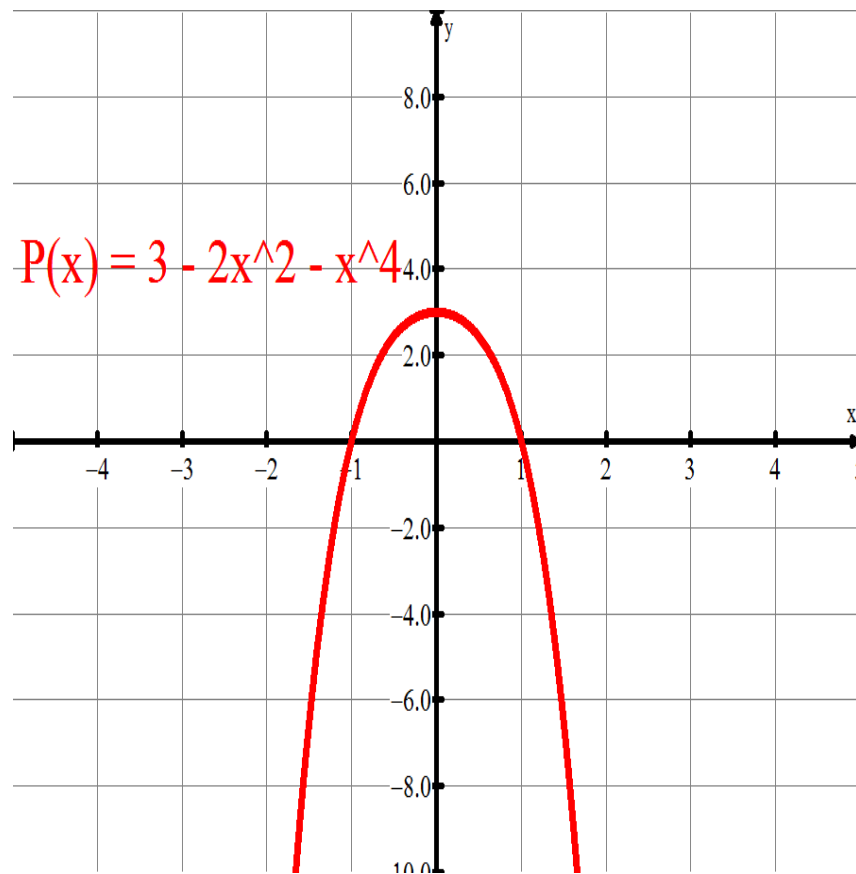
(B) Solving if $x \in \mathbf{C}$ – Graphic Connection

- With $P(x) = 3 - 2x^2 - x^4$, we can now consider a graphic connection, given that

$$P(x) = -(x^2 + 3)(x - 1)(x + 1)$$

or given that

$$P(x) = -(x - 1)(x + 1)(x - i\sqrt{3})(x + i\sqrt{3})$$



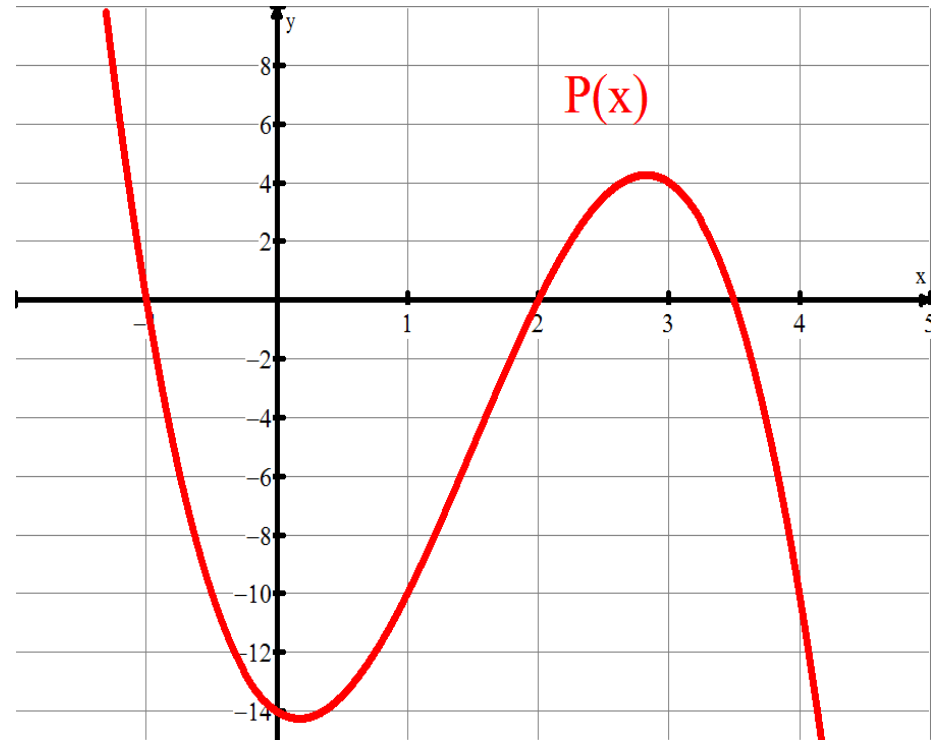
(C) Fundamental Theorem of Algebra

- The fundamental theorem of algebra can be stated in many ways:
 - (a) If $P(x)$ is a polynomial of degree n then $P(x)$ will have exactly n zeroes (real or complex), some of which may repeat.
 - (b) Every polynomial function of degree $n \geq 1$ has exactly n complex zeroes, counting multiplicities
 - (c) If $P(x)$ has a nonreal root, $a+bi$, where $b \neq 0$, then its conjugate, $a-bi$ is also a root
 - (d) Every polynomial can be factored (over the real numbers) into a product of linear factors and irreducible quadratic factors
- What does it all mean → we can solve EVERY polynomial (it may be REALLY difficult, but it can be done!)

(D) Using the FTA

- Write an equation of a polynomial whose roots are $x = 1$, $x = 2$ and $x = \frac{3}{4}$
- Write the equation of the polynomial whose roots are 1, -2, -4, & 6 and a point $(-1, -84)$
- Write the equation of a polynomial whose roots are $x = 2$ (with a multiplicity of 2) as well as $x = -1 \pm \sqrt{2}$

- Write the equation of a polynomial whose graph is given:



(D) Using the FTA

- Given that $1 - 3i$ is a root of $x^4 - 4x^3 + 13x^2 - 18x - 10 = 0$, find the remaining roots.
- Write an equation of a third degree polynomial whose given roots are 1 and i . Additionally, the polynomial passes through $(0,5)$
- Write the equation of a quartic wherein you know that one root is $2 - i$ and that the root $x = 3$ has a multiplicity of 2.

(E) Further Examples

- The equation $x^3 - 3x^2 - 10x + 24 = 0$ has roots of 2, h , and k . Determine a quadratic equation whose roots are $h - k$ and hk .
- The 5th degree polynomial, $f(x)$, is divisible by x^3 and $f(x) - 1$ is divisible by $(x - 1)^3$. Find $f(x)$.
- Find the polynomial $p(x)$ with integer coefficients such that one solution of the equation $p(x)=0$ is $1 + \sqrt{2} + \sqrt{3}$.

Homework

- Textbook, S7.5, p463-464,
Q17,19,27,28,31,32,38,43,45,46,48,49,50
- Do some with & some without the TI-84