

Lesson 20 – Algebra of Polynomials – Intro to Factoring

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Box Data

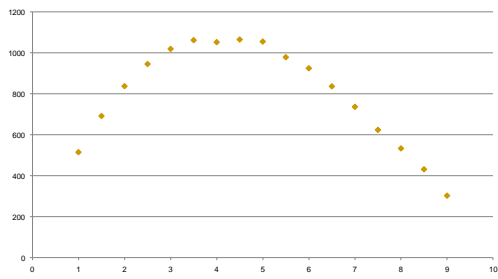
1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9
515	692	838	946			1053		1066	979	926	837	737	624	533	431	302

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Box Data

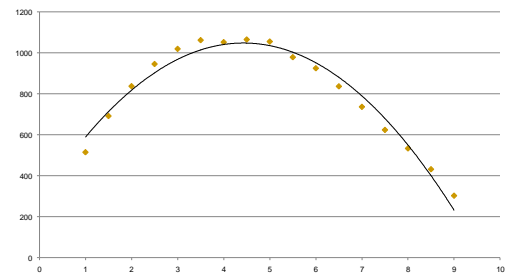


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Box Data - Quadratic

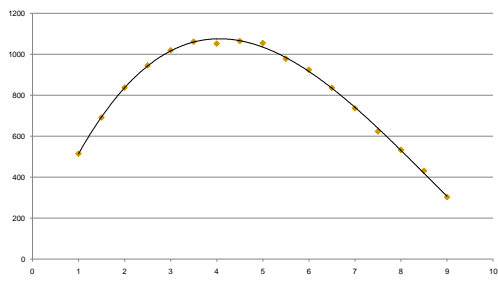


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Box Data - Cubic

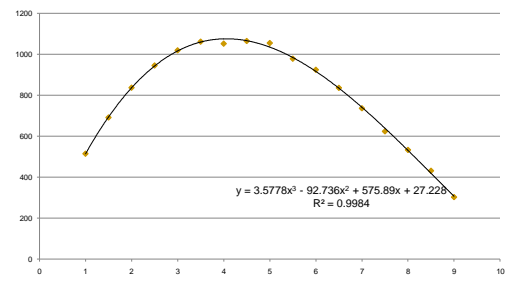


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Box Data – Cubic with Formula & R²



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Questions About our Boxes

- 1. Prepare a scatterplot on your GDC
- 2. Explain WHY a quadratic equation CANNOT be used to model the equation (from scatterplot/regression & geometric perspective)
- 3. Determine an equation to model the relationship between square width (x) and volume. Express in factored form and in standard form

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Questions About our Boxes

- (4) Determine the domain and range of the function and explain each in the context of the boxes
- (5) Determine the x -intercepts and explain each in the context of the boxes
- (6) Evaluate $V(4.25)$ and each in the context of the boxes
- (7) Solve $V^{-1}(x) = 6.33333\dots$ explain in the context of the boxes
- (8) Solve $400 = V(x)$ explain in the context of the boxes

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Questions About our Boxes

- (9) On what interval is the $V(x)$ function increasing. Explain in the context of the boxes
- (10) Determine & classify all extrema of $V(x)$ given your restricted domain from $Q(4)$ and explain in the context of the boxes

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Lesson Objectives

- Understand the similarities and differences between the algebraic processes involving second degree polynomials (quadratics) and higher order polynomials (cubics, quartics)
- Understand that factoring involves division and the connections between the 2 processes
- Mastery of the factoring of polynomials using the algebraic processes of long & synthetic division
- Introduce the remainder and rational root theorems and apply them to factor polynomials
- Reinforce the understanding of the connection between factors and roots

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(A) Why do algebra with polynomials?

- In our work with quadratic polynomials, we focused on the algebraic analysis of quadratics \rightarrow WHY \rightarrow we were interested in finding either (i) points on the parabola, (ii) the extrema (vertex) or (iii) zeroes, x -intercepts, roots
- What we saw was that quadratics can be presented in three forms (standard, vertex, factored) \rightarrow from which we can perform "simple" algebra in order to determine the (i) points on the parabola, (ii) the extrema (vertex) or (iii) zeroes, x -intercepts, roots

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(A) Why do algebra with polynomials?

- But now that we are consider further polynomials (cubics, quartics) \rightarrow do the same "simple" algebraic processes apply??? \rightarrow NO!!
- Through "simple" algebraic processes with polynomials, we can only "easily" determine the (i) points on the parabola, and (ii) zeroes, x -intercepts, roots
- To find the extrema (max,min), we need other algebraic processes \rightarrow CALCULUS

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(B) Algebra of Polynomials → Evaluation of Polynomials

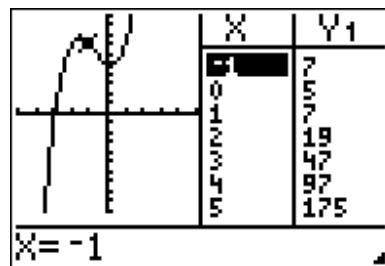
- It is relatively easy to EVALUATE any function → simply substitute in a value for the variable
- i.e if $P(x) = x^3 + 2x^2 - x + 5$, evaluate $P(-1)$
- so $P(-1) = (-1)^3 + 2(-1)^2 - (-1) + 5 = -1 + 2 + 1 + 5 = 7$
- So $P(-1) = 7$ → GRAPHIC/NUMERIC connection → it means that we have the order pair/point $(-1, 7)$ on our table of values and on our graph

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(B) Algebra of Polynomials → Evaluation of Polynomials → Graphic & Numeric Connection



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(B) Algebra of Polynomials → Evaluation of Polynomials → examples

- Evaluate $4x^3 - 6x^2 + x - 3$ for $x = 2$
- Evaluate $P(-4)$ if $P(x) = 2x^3 + 16x^2 + 23x - 36$ (significance?????)
- Evaluate $x^2 + 6x^3 - 5$ for $x = \frac{1}{2}$
- Evaluate $P(2)$ if $P(x) = x^4 + 4x^3 + 2x^2 - 3x - 50$

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(C) Roots & Factors

- In our work with quadratics, we saw the "factored" form or "intercept" form of a quadratic equation/expression
- i.e. $f(x) = x^2 - x - 2 = (x - 2)(x + 1)$ → factored form of eqn
- So when we solve $f(x) = 0 \rightarrow 0 = (x - 2)(x + 1)$, we saw that the zeroes/x-intercepts/roots were $x = 2$ and $x = -1$
- So we established the following connection:
 - Factors → $(x - 2)$ and $(x + 1)$
 - Roots → $x = 2$ and $x = -1$
- So we will now reiterate the following connections:
 - If $(x - R)$ is a **factor** of $P(x)$, then $x = R$ is **root** of $P(x)$ AND THE CONVERSE
 - If $x = R$ is a **root** of $P(x)$, then $(x - R)$ is a **factor** of $P(x)$

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(D) Factoring → The Process

- So how do we factor in general?
- Factor the number 38,754 → $(2)(3)(3)(2153)$
- KEY IDEA → the PROCESS that we used was DIVISION

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(E) Division of Polynomials

- So, in order to factor, we need to be able to DIVIDE
- Q → HOW do we divide polynomials??
- We will show you 2 ways to divide polynomials → long division & synthetic division
- Keep in mind WHY we are factoring the polynomials → I will eventually ask for ROOTS

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(E) Division of Polynomials

- Divide using LD
- Divide using SD:

$$3x^3 + 13x^2 - 9x + 6 \text{ by } x + 5$$

$$x+5 \overline{) 3x^3 + 13x^2 - 9x + 6}$$

$$3x^3 + 13x^2 - 9x + 6 \text{ by } x + 5$$

$$\begin{array}{r} -5 \overline{) 3 \ 13 \ -9 \ 6} \\ \underline{ 15 } \\ 13 \\ 15 \\ 13 \\ 15 \\ 13 \\ 15 \\ 13 \end{array}$$

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(F) Practice - Examples

- Show examples 1,2,3 using both division methods
- ex 1. Divide $x^3 - 42x + 30$ by $x - 6$
- ex 2. Divide $x^2 + 6x^3 - 5$ by $2x - 1$
- ex 3. Divide $x^4 + 4x^3 + 2x^2 - 3x - 50$ by $x - 2$

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(G) Division & Factoring Connection

- Recall → WHY are we dividing?
- ANS → to see if the divisor is a FACTOR of the dividend
- WHAT do we look for when we do the division to answer the "factor" question → the remainder!!
- Is 4 a factor of 72? How do you know??
- Is 5 a factor of 72? How do you know??

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(H) Factoring Polynomials – An Example

- So we have in place a method for factoring polynomials → use division to see if the remainder is 0!!
- Is $(x + 4)$ a factor of $P(x) = 2x^2 + 7x + 3$

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(H) Factoring Polynomials – An Example

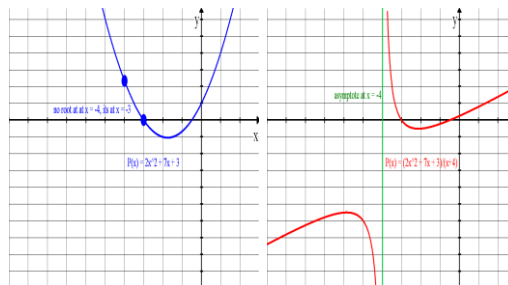
- When dividing $(2x^2 + 7x + 3)$ by $(x + 4)$ and we get $2x - 1$ with a remainder of 4
- Conclusions to be made
 - (i) $x + 4$ is not a factor of $2x^2 + 7x + 3$
 - (ii) $x + 4$ does not divide evenly into $2x^2 + 7x + 3$
 - (iii) when $2x^2 + 7x + 3$ is divided by $x + 4$, the remainder is 7
 - (iv) $2x^2 + 7x + 3 = (x + 4)(2x - 1) + 7$
 - (v) $(2x^2 + 7x + 3)/(x + 4) = 2x - 1 + 7/(x + 4)$
- Show on GC and make connections
 - (i) graph $2x^2 + 7x + 3$ and see that $x = -4$ is not a root
 - (ii) graph $(2x^2 + 7x + 3)/(x + 4)$ → and we see???

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(H) Factoring Polynomials – An Example



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(H) Factoring Polynomials – An Example

- So we have in place a method for factoring polynomials → use division to see if the remainder is 0!!
- Q? Is $x + 5$ a factor of $3x^3 + 13x^2 - 9x + 6$??
- How do you know?
- Q? Is $x + 5$ a factor of $3x^3 + 13x^2 - 9x + 5$??
- How do you know?

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(H) Factoring Polynomials – An Example

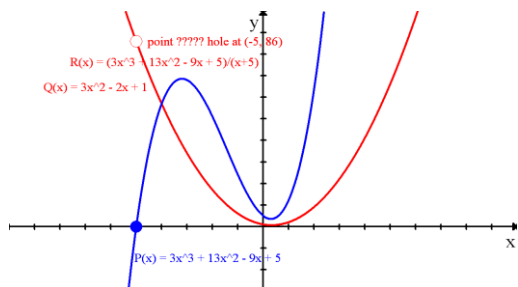
- Divide $3x^3 + 13x^2 - 9x + 5$ by $x + 5$
- conclusions to be made:* - all 5 conclusions are equivalent and say mean the same thing
 - $x + 5$ is a factor of $3x^3 + 13x^2 - 9x + 5$
 - $x + 5$ divides evenly into $3x^3 + 13x^2 - 9x + 5$
 - when $3x^3 + 13x^2 - 9x + 5$ is divided by $x + 5$, there is no remainder
 - $3x^3 + 13x^2 - 9x + 5 = (x + 5)(3x^2 - 2x + 1)$
 - $(3x^3 + 13x^2 - 9x + 5) / (x + 5) = 3x^2 - 2x + 1$
- Show on GC and make connections
 - graph $3x^3 + 13x^2 - 9x + 5$ and see that $x = -5$ is a root or a zero or an x-intercept
 - graph $(3x^3 + 13x^2 - 9x + 5) / (x + 5) →$ and we see??

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(H) Factoring Polynomials – An Example



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(I) Factoring Polynomials – Practice

- Factor $g(x) = x^3 + 2x^2 - 16x - 32$
- Factor $P(x) = x^3 - 3x^2 - 2x + 6$
- Factor $f(x) = x^3 - x^2 - 14x + 24$
- Factor $P(x) = x^3 - 6x^2 + 13x - 20$
- Factor $y = x^3 + 4x^2 + 7x + 6$
- Factor $y = x^3 - 9x^2 + 24x - 16$
- Factor $f(x) = x^4 + x^3 - 11x^2 - 9x + 18$
- Factor $g(x) = x^4 - 3x^3 + 6x^2 - 2x - 12$

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(J) Factoring Polynomials – The Remainder Theorem

- So in factoring $P(x)$, we use the remainder of the division in order to make a decision about whether or not the $x - A$ is/is not a factor of $P(x)$
- So if we only want to find a remainder, is there another way (rather than only division?)
- YES there is → it's called the Remainder Theorem

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(J) Factoring Polynomials – The Remainder Theorem

- Divide $-2x^3 + 3x^2 - 39x - 20$ by $x + 1$
- Evaluate $P(-1)$. What do you notice?
- What must be true about $x + 1$
- Divide $x^3 - 8x^2 + 11x + 5$ by $x - 2$
- Evaluate $P(2)$. What do you notice?
- What must be true about $(x - 2)$?
- Divide $3x^3 - 4x^2 - 2x - 5$ by $x + 1$
- Evaluate $P(-1)$. What do you notice?
- EXPLAIN WHY THIS WORKS????

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(J) Factoring Polynomials – The Remainder Theorem

- Divide $3x^3 - 4x^2 - 2x - 5$ by $x + 1$
- $Q(x) = (3x^2 - 7x + 5)$ with a remainder of -10
- Evaluate $P(-1) \rightarrow$ equals -10
- EXPLAIN WHY THIS WORKS????
- if rewritten as:
 - $3x^3 - 4x^2 - 2x - 5 = (x + 1)(3x^2 - 7x + 5) - 10$,
 - $P(-1) = (-1 + 1)(3(-1)^2 - 7(-1) + 5) - 10 = -10$

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(J) Factoring Polynomials – The Remainder Theorem

- the remainder theorem states "when a polynomial, $P(x)$, is divided by $(ax - b)$, and the remainder contains *no term in x*, then the remainder is equal to $P(b/a)$
- PROVE WHY THIS IS TRUE !?!?!?!?

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(K) Factoring Polynomials – The Remainder Theorem - Examples

- Factor the following polynomials using the Remainder Theorem:
 - $P(x) = -2x^3 - x^2 + 25x - 12$
 - $P(x) = 4x^3 - 12x^2 - 19x + 12$
 - $P(x) = 12x^4 + 32x^3 - 15x^2 - 8x + 3$

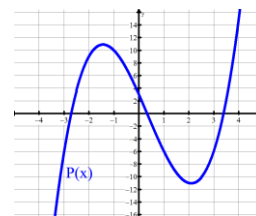
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(K) Factoring Polynomials – The Remainder Theorem - Connection

- So, from the graph on the right, determine the remainder when $P(x)$ is divided by:
 - (i) $x + 2$
 - (ii) $x - 2$
 - (iii) $x - 4$



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(K) Factoring Polynomials – The Remainder Theorem - Examples

- Find k so that when $x^2 + 8x + k$ is divided by $x - 2$, the remainder is 3
- Find the value of k so that when $x^3 + 5x^2 + 6x + 11$ is divided by $x + k$, the remainder is 3
- When $P(x) = ax^3 - x^2 - x + b$ is divided by $x - 1$, the remainder is 6. When $P(x)$ is divided by $x + 2$, the remainder is 9. What are the values of a and b ?
- Use the remainder theorem to determine if $(x - 4)$ is a factor of $P(x)$ if $P(x) = x^4 - 16x^2 - 2x + 6$

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(L) Factoring Polynomials – the Rational Root Theorem

- Our previous examples were slightly misleading ... as in too easy (leading coefficient was deliberately 1)
- Consider this example $\rightarrow -12x^3 + 20x^2 + 33x - 20$ which when factored becomes $(2x-1)(3x+4)(5-2x)$ so the roots would be $\frac{1}{2}$, $-4/3$, and $5/2$
- Make the following observation \rightarrow that the numerator of the roots (1, -4, 5) are factors of the constant term (-20) while the denominator of the roots (2,3,2) are factors of the leading coefficient (-12)
- We can test this idea with other polynomials \rightarrow we will find the same pattern \rightarrow that the roots are in fact some combination of the factors of the leading coefficient and the constant term

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(L) Factoring Polynomials – the Rational Root Theorem

- Our previous observation (although limited in development) leads to the following theorem:
- Given $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$, if $P(x) = 0$ has a rational root of the form a/b and a/b is in lowest terms, then a must be a divisor of a_0 and b must be a divisor of a_n

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(L) Factoring Polynomials – the Rational Root Theorem

- So what does this theorem mean?
- If we want to factor $P(x) = 2x^3 - 5x^2 + 22x - 10$, then we first need to find a value a/b such that $P(a/b) = 0$
- So the factors of the leading coefficient are $\{\pm 1, \pm 2\}$ which are then the possible values for b
- The factors of the constant term, -10 , are $\{\pm 1, \pm 2, \pm 5, \pm 10\}$ which are then the possible values for a
- Thus the possible ratios a/b which we can test to find the factors are $\{\pm 1, \pm \frac{1}{2}, \pm 2, \pm \frac{5}{2}, \pm 5, \pm 10\}$
- As it then turns out, $P(\frac{1}{2})$ turns out to give $P(x) = 0$, meaning that $(x - \frac{1}{2})$ or $(2x - 1)$ is a factor of $P(x)$

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(M) Factoring Polynomials – the Rational Root Theorem - Examples

- Ex 1. To factor $P(x) = 2x^3 - 9x^2 + 7x + 6$, what values of x could you test according to the RRT
- Now factor $P(x)$
- Ex 2. To factor $P(x) = 3x^3 - 7x^2 + 8x - 2$ what values of x could you test according to the RRT
- Now factor $P(x)$
- ex 3 → Graph $f(x) = 3x^3 + x^2 - 22x - 24$ using intercepts, points, and end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease.

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(N) Conclusion

- So, we now have some “simple” algebra tools that we can use to factor polynomials
- We use the rational root theorem and the remainder theorem
- We use these techniques in order to determine whether a chosen binomial $(ax + b)$ is or is not a factor of our polynomial

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Homework

- Homework:
- Ex. 7.3 (p. 429) # 26-56 evens, 62-65, 73-83 odds, 87, 93, 97, 99

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