

Lesson 19 – Investigating Polynomial Functions

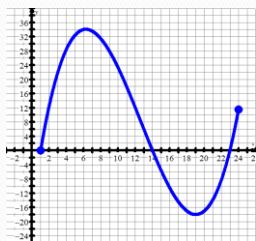
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Lesson Objectives

- Describe the key features of the graph of a polynomial function given in root form & standard form
- Graph a cubic, and quartic function on a graphing calculator
- Use the GDC to analyze the features of the polynomial function

Modelling with Cubic Fcns

- My brother's business has the following graph which represents his profits for the years 2008 & 2009.
- Answer the following questions:

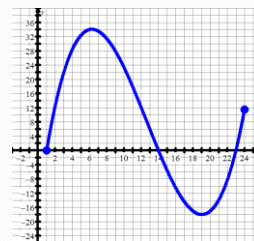


Cubic modelling

$$y = 0.05(x-1)(x-14)(x-23)$$

Modeling with Cubic Fcns

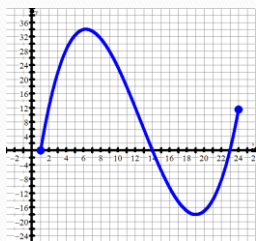
- Answer the following questions:
- (a) When were the profits increasing?
- (b) When were the profits decreasing?
- (c) when did his company maximize its profits?
- (d) When did the company minimize its profits?
- (e) When did the company make no profit?



$$y = 0.05(x-1)(x-14)(x-23)$$

Modeling with Cubic Fcns

- Answer the following questions:
- (a) When did the profit exceed \$20,000
- (b) Does this fcn work when predicting his profits for the next 2 years? Why/why not?
- (c) when did his company revenues exceed its expenses?
- (d) where are the local/global extrema & classify
- (e) Estimate/evaluate $y(3)$ and interpret



$$y = 0.05(x-1)(x-14)(x-23)$$

(A) Terminology

- Polynomials:** an expression in the form of $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$ where a_0, a_1, \dots, a_n are real numbers and n is a natural number
- Polynomial Functions:** a function whose equation is defined by a polynomial in one variable.
ex: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$
- leading coefficient:** the coefficient of the term with the highest power
- degree:** the value of the highest exponent on the variable
- Standard Form:** the function is expressed such that the terms are written in descending order of the exponents

(A) Terminology

- ▶ **Domain:** the set of all possible x values (independent variable) in a function
- ▶ **Range:** the set of all possible function values (dependent variable, or y values)
- ▶ **to evaluate a function:** substituting in a value for the variable and then determining a function value. Ex $f(3)$
- ▶ **zeroes, roots, x-Intercepts:** where the function crosses the x axes
- ▶ **y-intercepts:** where the function crosses the y axes
- ▶ **direction of opening:** parts of the curve opens up or down
- ▶ **symmetry:** whether the graph of the function has "halves" which are mirror images of each other

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(A) Terminology

- ▶ **turning point:** points where the direction of the function changes
- ▶ **maximum:** the highest point on a function
- ▶ **minimum:** the lowest point on a function
- ▶ **local vs absolute:** a max can be a highest point in the entire domain (absolute) or only over a specified region within the domain (local). Likewise for a minimum.
- ▶ **Increase:** the part of the domain (the interval) where the function values are getting larger as the independent variable gets higher; if $f(x_1) < f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the right (or down to the left)
- ▶ **decrease:** the part of the domain (the interval) where the function values are getting smaller as the independent variable gets higher; if $f(x_1) > f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the left (or down to the right)
- ▶ **"end behaviour":** describing the function values (or appearance of the graph) as x values getting infinitely large positively or infinitely large negatively

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(B) Types of Polynomial Functions

- ▶ (i) **Linear Functions** that generate graphs of straight lines, polynomials of degree one $\Rightarrow f(x) = a_1x_1 + a_0$ or more commonly written as $y = mx + b$, or $Ax + By + C = 0$, or $y = k(x - s)$
- ▶ (ii) **Quadratic Functions** that generate graphs of parabolas; polynomials of degree two $\rightarrow f(x) = a_2x^2 + a_1x^1 + a_0$ or $y = Ax^2 + Bx + C$ or $y = a(x-s)(x-t)$ or $y = a(x-h)^2 + k$
- ▶ (iii) **Cubic** polynomials of degree 3
- ▶ (iv) **Quartic:** polynomials of degree 4

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(C) Investigating Characteristics of Polynomial Functions

- ▶ We can complete the following analysis for polynomials of degrees 3 through 5 and then make some generalizations or summaries:
- ▶ In order to carry out this investigation, use a GDC
- ▶ [You may also use the following program from AnalyzeMath](#)

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(i) Cubic Functions

- ▶ For the cubic functions, determine the following:
 - ▶ $f(x) = x^3 - 5x^2 + 3x + 4$
 - ▶ $f(x) = -2x^3 + 8x^2 - 5x + 3$
 - ▶ $f(x) = -3x^3 - 15x^2 - 9x + 27$
- ▶ (1) Leading coefficient (2) degree
- ▶ (3) domain and range (4) evaluating $f(-2)$
- ▶ (5) zeroes or roots (6) y-intercept
- ▶ (7) Symmetry (8) turning points
- ▶ (9) maximum values (local and absolute)
- ▶ (10) minimum values (local and absolute)
- ▶ (11) intervals of increase and intervals of decrease
- ▶ (12) end behaviour (+x) and end behaviour (-x)

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Conclusions for Cubic Functions

1. Describe the general shape of a cubic function
2. Describe how the graph of a cubic function with a positive leading coefficient is different than a cubic with a negative leading coefficient
3. What does a_0 represent on the graph of a cubic?
4. How many real roots do/can cubic functions have?
5. How many complex roots do/can cubic functions have?

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Conclusions for Cubic Functions

- ▶ 6. How many turning points do/can cubic functions have?
- ▶ 7. How many intervals of increase do/can cubic functions have?
- ▶ 8. How many intervals of decrease do/can cubic functions have?
- ▶ 9. Describe the end behaviour ($\pm x$) of a cubic with a (i) positive (ii) negative leading coefficient
- ▶ 10. Are cubic functions symmetrical? (You may need to investigate further)

(ii) Quartic Functions

- ▶ For the quartic functions, determine the following:
 - ▶ $f(x) = -2x^4 - 4x^3 + 3x^2 + 6x + 9$
 - ▶ $f(x) = x^4 - 3x^3 + 3x^2 + 8x + 5$
 - ▶ $f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 + x + 1$
- ▶ (1) Leading coefficient (2) degree
- ▶ (3) domain and range (4) evaluating $f(-2)$
- ▶ (5) zeroes or roots (6) y-intercept
- ▶ (7) Symmetry (8) turning points
- ▶ (9) maximum values (local and absolute)
- ▶ (10) minimum values (local and absolute)
- ▶ (11) intervals of increase and intervals of decrease
- ▶ (12) end behaviour ($+x$) and end behaviour ($-x$)

Conclusions for Quartic Functions

- ▶ 1. Describe the general shape of a quartic function
- ▶ 2. Describe how the graph of a quartic function with a positive leading coefficient is different than a quartic with a negative leading coefficient
- ▶ 3. What does a_0 represent on the graph of a quartic?
- ▶ 4. How many real roots do/can quartic functions have?
- ▶ 5. How many complex roots do/can quartic functions have?

Conclusions for Quartic Functions

- ▶ 6. How many turning points do/can quartic functions have?
- ▶ 7. How many intervals of increase do/can quartic functions have?
- ▶ 8. How many intervals of decrease do/can quartic functions have?
- ▶ 9. Describe the end behaviour ($\pm x$) of a quartic with a (i) positive (ii) negative leading coefficient
- ▶ 10. Are quartic functions symmetrical? (You may need to investigate further)

(D) Examples of Algebraic Work with Polynomial Functions

- ▶ ex 1. Expand & simplify
 $h(x) = (x-1)(x+3)^2(x+2)$.
- ▶ ex 2. Where are the zeroes of $h(x)$?
- ▶ ex 3. Predict the end behaviour of $h(x)$.
- ▶ ex 4. Predict the shape/appearance of $h(x)$.
- ▶ ex 5. Use a table of values to find additional points on $h(x)$ and sketch a graph.
- ▶ ex 6. Predict the intervals of increase and decrease for $h(x)$.
- ▶ ex 7. Estimate where the turning points of $h(x)$ are. Are the max/min? and local/absolute if domain was $[-4, 1]$

(D) Examples of Algebraic Work with Polynomial Functions

- ▶ ex 8. Sketch a graph of the polynomial function which has a degree of 4, a negative leading coefficient, 3 zeroes and 3 turning points
- ▶ ex 9. Equation writing: Determine the equation of a cubic whose roots are $-2, 3, 4$ and $f(5) = 28$
- ▶ ex 10. Prepare a table of differences for $f(x) = -2x^3 + 4x^2 - 3x - 2$. What is the constant difference and when does it occur? Is there a relationship between the equation and the constant difference? Can you predict the constant difference for $g(x) = 4x^4 + x^3 - x^2 + 4x - 5$?

(E) Internet Links

- ▶ [Polynomial functions from The Math Page](#)
- ▶ [Polynomial Functions from Calculus Quest](#)
- ▶ [Polynomial Tutorial from WTAMU](#)
- ▶ [Polynomial Functions from AnalyzeMath](#)

(E) Homework

- ▶ Ex. 7.1 (p. 429) # 13–21 odds, 26–36 evens, 45–57 odds, 62,63
- ▶ Ex. 7.2 (p. 438) # 13–23 odds, 29–37 odds