Lesson 17 – Solving Quadratic Inequalities

Math 2 Honors – Santowski

FAST FIVE

- Solve the following question graphically:
- If the revenue (in millions of dollars) for a company is given by the equation $R(x) = -3x^2 + 26x$, where x is the number of items made (in thousands) and the expenses (in millions of dollars) are given by the equation $C(x) = 8x + 18$, determine the number of items that should be produced so that the company has a profit greater than zero.

Lesson Objectives

- Write, solve, and graph a quadratic inequality in one variable
- Explore various methods for solving inequalities
- Apply inequalities with quadratics to modeling problems
- Write, solve, and graph a quadratic inequality in two variables

Solving Inequalities → Strategies

- We will highlight several strategies to use when solving inequalities:
- (a) Algebraic with inequalities
- (b) Numerically with Sign charts
- (c) graphical

(A) Strategy #1 – Algebraic – Zero Product Property

- Recall the zero product property → if the product of two numbers is zero, then either (or both) of the numbers must be a zero
- In mathematical symbols, if $ab = 0$, then $a = 0$ or/and $b = 0$
- So how does this apply (if it indeed does) to an inequality → if $ab > 0$, then .....?? or alternatively, if $ab <$ o, then ...... ???
- So what must be true of a and b in this inequalities?

(A) Strategy #1 – Algebraic – Zero Product Property

- Let's think about the statement $ab < 0$
- We are trying to think about two numbers that are being multiplied together, such that their product is less than zero → or that their product is negative
- This negative product happens if (i) either $a < 0$ and at the same time $b > 0$ (or rather if a is negative and b is positive) or (ii) $b < 0$ and at the same time $a < 0$ (or if b is negative and at the same time a is positive)

(A) Strategy #1 – Algebraic – Zero Product Property

- Let's see how this works ➔ Solve $(x + 2)(x - 1) < 0$
- So one of two conditions are true:
  - (i) $(x + 2)>0$ and $(x - 1)<0$
  - So we have $x > -2$ and $x < 1$ ➔ How can BOTH these be true ➔ only if $-2 < x < 1$ ➔ set up a number line to show
  - (ii) $(x + 2)<0$ and $(x - 1)>0$
  - So we have $x < -2$ and $x > 1$ ➔ How can BOTH these be true ➔ it can't!! ➔ set up a number line to show
- So there we have our solution ➔ $(x + 2)(x - 1) < 0$ only if $-2 < x < 1$

(A) Strategy #1 – Algebraic – Zero Product Property

- Let's think about the statement $ab > 0$
- We are trying to think about two numbers that are being multiplied together, such that their product is more than zero ➔ or that their product is positive
- This positive product happens if either (i) $a > 0$ and at the same time $b > 0$ (or rather if a and b are positive) or (ii) $b < 0$ and at the same time $a < 0$ (or if a and b are both negative)

(A) Strategy #1 – Algebraic – Zero Product Property

- Now change it to ➔ Solve $(x + 2)(x - 1) > 0$
- So one of two conditions are true:
  - (i) $(x + 2)>0$ and $(x - 1)>0$
  - So we have $x > -2$ and $x > 1$ ➔ HOW can both these be true ➔ only if $x > 1$ ➔ set up a number line to show
  - (ii) $(x + 2)<0$ and $(x - 1)<0$
  - So we have $x < -2$ and $x < 1$ ➔ HOW can both these be true ➔ only if $x < -2$ ➔ set up a number line to show
- So there we have our solution ➔ $(x + 2)(x - 1) > 0$ only if $x < -2$ or $x > 1$

(B) Strategy #2 – Test Points

- Solve $x^2 + x - 2 < 0$
- We will factor ➔$(x + 2)(x - 1) < 0$
- So our key domain values will be $x = -2$ and $x = 1$ (WHY??)
- So let's divide our domain into three "sets" and use a test point in each set:
- (i) $x < -2$ ➔ test with $x = -4$ ➔ observe that .....
- (ii) $-2 < x < 1$ ➔ test with $x = 0$ ➔ observe that ...
- (iii) $x > 1$ ➔ test with $x = 2$ ➔ observe that .....

(B) Strategy #2 – Numeric – Sign Chart

- Show the solution to $x^2 + x - 2 > 0$ by means of a table/chart technique that takes into account the domain as it is divided into its three intervals (in this case)
- So again I'll factor $(x + 2)(x - 1) > 0$
- Then, I'll set up a sign chart as follows:

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- $(x + 2)(x - 1) > 0$
- the domain is divided into three intervals (in this case)
- Then, I'll set up a sign chart as follows:

	x < -2	-2<x<1	X > 1
(x + 2)	-ve	+ve	+ve
(x – 1)	-ve	-ve	+ve
Q(x)	+ve	-ve	+ve

Factored quadratic

Sign chart

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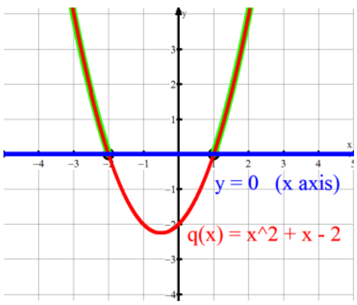
(C) Strategy #3 – Graphically

- Any inequality can be solved graphically, provided that we can generate the graph and then KNOW what we are looking for!
- So to solve $x^2 + x - 2 > 0$ (or $(x + 2)(x - 1) > 0$), we simply graph the system ➔ $\begin{cases} y = x^2 + x - 2 \\ y = 0 \end{cases}$

the quadratic and the line y = 0 (which happens to be the x-axis

- After we graph, what do we look for? ➔ in our case, look where the quadratic is > the line ➔ meaning where is the quadratic ABOVE the line!

(C) Strategy #3 – Graphically



(D) Inequalities & Completing the Square

- So how would we ALGEBRAICALLY work through the same question if we HAD to use the completing the square method
- So if $x^2 + x - 2 > 0$
- Then $(x^2 + x + ¼ - ¼) - 2 > 0$
- And $(x + ½)^2 - 9/4 > 0$
- And we finally get $(x + ½)^2 > 9/4$
- Now we can square root both sides (as the inverse operation of squaring)

(D) Inequalities & Completing the Square

- So this is where we need to be careful!!
- If $(x + ½)^2 > 9/4$
- We clearly know that the square root of 9/4 is 3/2!
- But what is the square root of a number/expression that is squared ➔ CLEARLY the possibilities for the square root of the number x + ½ are a positive number and also a negative number!!
- So how do we express the idea that the LS of our equation (our input) can be either a +ve or –ve, but yet return only a +3/2 as its output???? ➔ **absolute value!!**

(D) Inequalities & Completing the Square

- So from $(x + ½)^2 > 9/4$ ➔ we will write the next step of our solution as $|x + ½| > 3/2$
- Then $+(x + ½) > 3/2$ ➔ $x > 1$
- And $-(x + ½) > 3/2$ ➔ $x < -2$
- As we expected from our other three solutions!

Practice Questions

- Solve the following inequalities (CALC INACTIVE) and verify GRAPHICALLY (Using CALC):
- (a) $2x^2 - 14x > -20$ (factoring)
- (b) $x^2 + 2x - 5 \leq 0$ (c/s)
- (c) $-x^2 + 3x + 3 < 0$ (QF)
- (d) $\frac{1}{2}(x + 3)^2 - 7 \geq -1$

(E) Applications of Inequalities

- A rock is tossed into the air from a bridge over a river. Its height, *h* in meters, above the water after *t* seconds is $h(t) = -5(t - 2)^2 + 45$.
- (a) From what height above the water was the rock tossed?
- (b) Find the maximum height of the rock and the time when this maximum height is reached.
- (c) Is the rock still in the air after 4.5 seconds. Show work. Explain your answer.
- (d) When does the rock hit the water?
- (e) For how many seconds is the rock ABOVE 33.75 m?

(E) Applications of Inequalities

- The population of Mathopolis can be modeled by $P(t) = -0.5t^2 + 20t + 200$, where P is population in thousands and t is time in years from 1990 onward (i.e. $t = 0$ is the year 1990)
- (b) Find the population in the year 2003
- (c) When was the population over 350,000?

Homework

- p. 334 # 15,17, 25,27,28,29, 39,43,51, 58-64