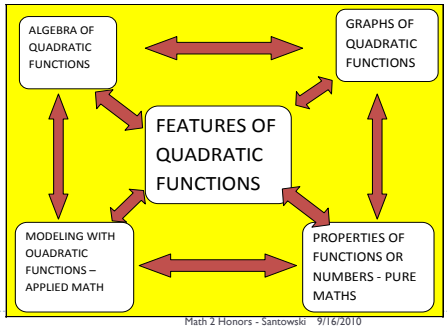
Lesson 13 – Algebra of Quadratic Functions – Completing the Square

Math 2 Honors - Santowski

BIG PICTURE



(A) Skills Review/Consolidation

- Identify which properties of real numbers are highlighted by the following statements:
- (1) 4 + 7 + 0 = 11 (2) 4 + 7 + 3 – 3 = 11
- What is a perfect square trinomial?
- What is the "key step/idea" in making the c/s process work?

(A) Skills Review/Consolidation

- Solve the following using the c/s method:
- (a) $0 = 2(x – 3)^2 – 32$ (b) $0 = -4x^2 + 10x - 3$
- Find the vertex of the parabola
- (a) $f(x) = 2x^2 + 12x + 5$ (b) $g(x) = -2x^2 - 10x + 15$

(B) Skills Extension

1. If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
2. If $y = -4x^2 + kx – 1$, determine the value(s) of k for which the maximum value of the function is an integer. Explain your reasoning
3. Given $f(x) = ax^2 + bx + c$, use the C/S method to rewrite the equation in vertex form, $f(x) = a(x – h)^2 + k$, and thereby determine h and k in terms of a,b & c
4. Use the C/S method to rewrite $f(x) = ax^2 + bx + c$ in factored form, $f(x) = a(x – R_1)(x – R_2)$, and thereby determine R_1 and R_2 in terms of a,b, & c.

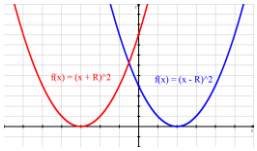
(B) Skills Extension

1. Find the minimum point of $y = x^2 – bx + 4$ using the c/s method
2. Find the maximum point of $y = c + 5x – x^2$ using the c/s method
3. The nonzero roots of the equation $3x^2 - 4x + k = 0$ are in the ratio 3:1. Determine the roots and the value of k.
4. The point (0, 0) is on the parabola $y = 5x - x^2$. What other point on the parabola has x and y coordinates that are equal?

(C) Graphing Connection

- (a) Graph $f(x) = (x + 4)^2$
- (b) Graph $g(x) = (5 - 2x)^2$

- (c) Graph $y = (x - R)^2$



(C) Graphing Connection

- Given the quadratic functions, change the equation to vertex form to determine the:

- (i) domain and range
- (iii) vertex
- (iv) maximum/minimum point
- (v) maximum/minimum value
- (vi) Find the x-intercepts of f(x)

- (a) $f(x) = 3x^2 - 30x + 1$ (b) $f(x) = -2x^2 + 5x - 3$

(D) Applications – Modeling with QF

- Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for $11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every $1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?

(D) Applications – Modeling with QF

- The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where h is the height in m and d is the horizontal distance of the ball in meters from the batter.
  - (a) What is the maximum height reached by the baseball?
  - (b) When would the ball land on the ground??
  - (c) How far from the ground is the ball when I release the pitch?
  - (d) How high above the ground is the ball when the ball reaches the batter if she stands 20m from the pitcher?

(D) Applications – Modeling with QF

- The Brick Bakery sells more bagels when it reduces its prices, but then its profit changes as a result. The function $P(x) = -1000(x - 0.55)^2 + 300$ models the baker's daily profit P in dollars, from selling bagels, as a function of x, the price of each bagel in dollars. The bakery wishes to maximize its profit.

- a) What is the domain of the function? Can x be negative? Explain.
- b) Evaluate the daily profit for selling bagels for $0.40 each.
- c) Evaluate the daily profit for selling bagels for $0.85 each.
- d) For what other unit price could bagels be sold to produce the same profit as
- selling them for $0.40? for $0.85? Explain graphically how you determined
- these unit prices. Include a sketch with a reasonable scale in your explanation.
- e) Give a reason for why the higher unit price of bagels does not produce a
- greater profit.
- f) What unit price for bagels should the bakery charge to maximize the profit? What maximum profit does this produce?

(D) Applications – Modeling with QF

- The perimeter of a rectangle is 36 in.

- a) Define variables l and w to represent the dimensions of the rectangle, and sketch and label the rectangle.
- b) Write an equation for the area of the rectangle in terms of l and w.
- c) Use the perimeter equation to write length in terms of width; then substitute this equation into the area equation to write the area in terms of width only.
- d) Sketch the A (w) function. (Hint: width is x and area is y). Use a window that shows the two x – intercepts and the maximum of the function, and state the window dimensions. What kind of function is the area versus width function?
- e) What is the maximum area of a rectangle whose perimeter is 36 in, and what are its dimensions?

(E) WHY???

- Why does C/S work? ➔ we are creating perfect square trinomials ➔ we are taking our sums/differences of different base functions and rearranging them into a single base function ➔ the squaring function

(E) WHY???

- Why do we use the process of "completing the square" in the first place?
- First we saw that by factoring a Quad Eqn, we can re-express a sum/difference expression as a product of linear factors ➔ KEY POINT here ➔ which then allows us to ISOLATE A BASE FCN so that we can "solve" for the variable (using the ZPP)
- But how can we isolate a base function if we CAN'T FACTOR an equation in the first place????

(E) WHY???

- So if we can't factor, we use the method of completing the square BECAUSE we have now isolated the BASE FUNCTION ➔ it happens this time to be a SQUARING function, but we know an INVERSE function that allows us to "isolate" our variable!!!

(E) WHY?

- So as an EXTENSION exercise:
- (A) simplify the equation $x^2 + 4y^2 + 4x - 8y - 8 = 0$
- (B) Produce a graph of the relation
- (C) Graph $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$
- (D) How do your two graphs compare?
- (E) What was the point of the c/s method?