

M2H Lesson 13 – Completing the Square

(A) Skills Review/Consolidation

- Identify which properties of real numbers are highlighted by the following statements:
 - $4 + 7 + 0 = 11$
 - $4 + 7 + 3 - 3 = 11$
- What is a perfect square trinomial?
- What is the “key step/idea” in making the c/s process work?
- Solve the following using the c/s method:
 - $0 = 2(x - 3)^2 - 32$
 - $0 = -4x^2 + 10x - 3$
- Find the vertex of the parabola
 - $f(x) = 2x^2 + 12x + 5$
 - $g(x) = -2x^2 - 10x + 15$

(B) Skill Extension

- If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
- Determine the relationship between b and c such that $x^2 + bx + c$ is a perfect square trinomial (HINT: $(x + d)^2$ or list examples and look for relationships)
- If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the maximum value of the function is an integer. Explain your reasoning
- Find the minimum point of $y = x^2 - bx + 4$ using the c/s method
- Find the maximum point of $y = c + 5x - x^2$ using the c/s method
- Given $f(x) = ax^2 + bx + c$, use the C/S method to rewrite the equation in vertex form, $f(x) = a(x - h)^2 + k$, and thereby determine h and k in terms of a, b & c
- Use the C/S method to rewrite $f(x) = ax^2 + bx + c$ in factored form, $f(x) = a(x - R_1)(x - R_2)$, and thereby determine R_1 and R_2 in terms of a, b , & c .
- The nonzero roots of the equation $3x^2 - 4x + k = 0$ are in the ratio 3:1. Determine the roots and the value of k .
- The point $(0, 0)$ is on the parabola $y = 5x - x^2$. What other point on the parabola has x and y coordinates that are equal?

(C) Graphing Connection – Factoring

- Graph the following functions: (a) $f(x) = (x + 4)^2$ (b) Graph $g(x) = (5 - 2x)^2$ (c) Graph $y = (x - R)^2$
- Given the quadratic functions (a) $f(x) = 3x^2 - 30x + 1$ (b) $f(x) = -2x^2 + 5x - 3$, change the equation to vertex form to determine the: (a) domain and range, (b) vertex & the max/min point & value, (c) the x -intercepts of $f(x)$ and (d) Sketch

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(D) Applications

17. Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for \$11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every \$1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?
18. The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where h is the height in m and d is the horizontal distance of the ball in meters from the batter.
- What is the maximum height reached by the baseball?
 - When would the ball land on the ground??
 - How far from the ground is the ball when I release the pitch?
 - How high above the ground is the ball when the ball reaches the batter if she stands 20m from the pitcher?
19. The Brick Bakery sells more bagels when it reduces its prices, but then its profit changes as a result. The function $P(x) = -1000(x - 0.55)^2 + 300$ models the baker's daily profit P in dollars, from selling bagels, as a function of x , the price of each bagel in dollars. The bakery wishes to maximize its profit.
- What is the domain of the function? Can x be negative? Explain.
 - Evaluate the daily profit for selling bagels for \$0.40 each.
 - Evaluate the daily profit for selling bagels for \$0.85 each.
 - For what other unit price could bagels be sold to produce the same profit as selling them for \$0.40? for \$0.85? Explain graphically how you determined these unit prices. Include a sketch with a reasonable scale in your explanation.
 - Give a reason for why the higher unit price of bagels does not produce a greater profit.
 - What unit price for bagels should the bakery charge to maximize the profit? What maximum profit does this produce?
20. The perimeter of a rectangle is 36 in.
- Define variables l and w to represent the dimensions of the rectangle, and sketch and label the rectangle.
 - Write an equation for the area of the rectangle in terms of l and w .
 - Use the perimeter equation to write length in terms of width; then substitute this equation into the area equation to write the area in terms of width only.
 - Sketch the $A(w)$ function. (Hint: width is x and area is y). Use a window that shows the two x – intercepts and the maximum of the function, and state the window dimensions. What kind of function is the area versus width function?
 - What is the maximum area of a rectangle whose perimeter is 36 in, and what are its dimensions?

(E) WHY C/S??

21. Why does it work? (P/S)
22. Why do we use the process of “completing the square” in the first place?