

## Lesson 12 – Algebra of Quadratic Functions - Factoring

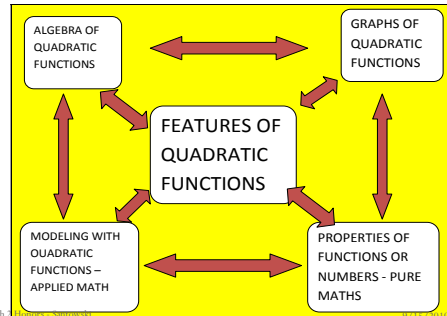
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1

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## BIG PICTURE



2

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### (A) Skills Review/Consolidation

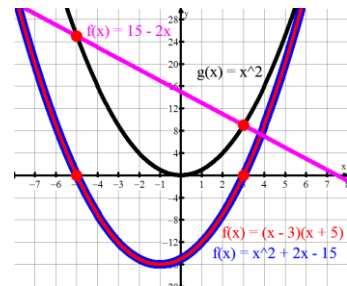
- Solve  $0 = 6x^2 + 23x + 7$
- ▶ Solve  $x^2 = 15 - 2x$
- ▶ Factor (a)  $4x^2 - 1$  (b)  $121 - 16x^2$
- ▶ Solve by factoring  $0 = 9x^2 - 1/9$
- ▶ Solve by factoring  $0 = 1/16x^2 - 3$
- ▶ Factor (a)  $x^2 - 8x + 16$  (b)  $4x^2 - 4x - 1$
- ▶ Solve the system defined by  $\begin{cases} y = x^2 \\ y = 15 - 2x \end{cases}$

3

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### (A) Skills Review/Consolidation



4

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### (B) Skill Extension

- Solve for  $b$  such that  $1/16x^2 - bx + 3$  is a perfect square trinomial
- Determine the relationship between  $b$  and  $c$  such that  $x^2 + bx + c$  is a perfect square trinomial (HINT:  $(x + d)^2$  or list examples and look for relationships)
- If the roots of a quadratic function are 5 and -2, can you determine an equation for the quadratic function? If so, what is the equation? If not, why not?

5

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### (B) Skill Extension

- In a quadratic equation with leading coefficient 1, Mr S reads the coefficient 16 of  $x$  wrongly as 19 and obtains the roots -15 and -4. Which of the following are the correct roots of the equation?
- If the equations  $x^2 - 6x + 5$  and  $Ax^2 + Bx + 1$  have the same roots, find the value of  $A + B$
- Is the quadratic  $x^2 - x - 6$  divisible by  $x - 3$ ? Explain your answer.

6

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## (B) Skill Extension

- If  $kx^2 - kx - 6$  is divisible by both  $x + 1$  and  $x + m$ , find the values of  $m$ .
- Find the possible values for  $p$  such that  $3x^2 + px + 5$  can be factored as a product of two first degree factors with integer coefficients
- Determine all values of  $t$  such that all roots of  $t(x - 1)(x - 2) = x$  are real

7

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## (C) Graphing Connection – Factoring

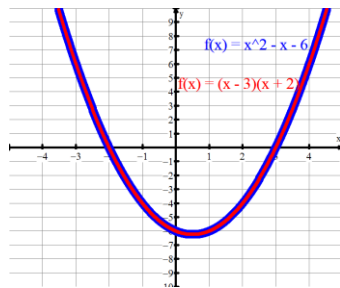
- ▶ So we can factor → what's the point?
- ▶ Now consider the expressions as functions
- ▶ Now  $x^2 - x - 6$  becomes  $f(x) = x^2 - x - 6$
- ▶ Now we can graph  $f(x) = x^2 - x - 6$
- ▶ And we can graph  $f(x) = (x - 3)(x + 2)$
- ▶ So we have the two forms of a quadratic equations (standard & factored) → So what?

8

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## (C) Graphing Connection – Factoring



9

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## (C) Graphing Connection – Factoring

- ▶ So we have the two forms of a quadratic equations (standard & factored) →  $f(x) = x^2 - x - 6 = (x - 3)(x + 2)$
- ▶ So we find the ROOTS/ZEROES/X-INTERCEPTS
- ▶ Now determine the equation of the axis of symmetry
- ▶ Now determine the vertex
- ▶ Now determine the y-intercept
- ▶ Now determine  $f(4)$
- ▶ Now sketch the parabola

10

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## (C) Graphing Connection – Factoring

- ▶ For the equations of parabolas listed below, determine the:
  - ▶ (a)  $f(x) = 2x^2 - 2x - 60$     (b)  $f(x) = 3x^2 - 7x - 6$
  - ▶ So we find the ROOTS/ZEROES/X-INTERCEPTS
  - ▶ Now determine the equation of the axis of symmetry
  - ▶ Now determine the vertex
  - ▶ Now determine the y-intercept
  - ▶ Now determine  $f(4)$
  - ▶ Now sketch the parabola

11

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## (C) Graphing Connection – Factoring

- EXTENSION QUESTIONS:
  - Determine the range of the parabola defined by  $f(x) = (x - 2)(x - R)$
  - What is the graphical significance of a perfect square trinomial?
  - What is the graphical significance a difference of squares trinomial?

12

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### (D) Applications

- A projectile whose height,  $h(t)$  in meters, varies with time,  $t$  in seconds, is modelled by:  $h(t) = -5t^2 + 15t + 50$
- Determine the flight time of the projectile?
- What is a reasonable domain for this application?
- Determine the maximum height of the projectile?
- When does the projectile reach its maximum height?
- When does the projectile reach a height of 60m?

13

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### (D) Applications

- Sasha wants to build a walkway of uniform width around a rectangular flower bed that measures 20m x 30m. Her budget is \$6000 and it will cost her \$10/m<sup>2</sup> to construct the path. How wide will the walkway be?

14

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### (E) WHY Factoring Works

- State the Zero Product Rule.
- EXPLAIN why the Zero Product Rule is important.

15

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### (E) WHY Factoring Works

- State the Zero Product Rule
- If the product of two numbers is 0, then it must follow that ..... ???
- Mathematically, if  $ab = 0$ , then .....
- So, if  $(x - r_1)(x - r_2) = 0$ , then .....
- But what if  $ab = 6$  → does that then mean that  $a = 6$  or  $b = 6$ ?

16

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### (G) WHY Factor in the First Place

- If  $f(x) = 2x$ , solve  $f(x) = 8$
- EXPLAIN the mathematical processes that are going on as you develop your solution → WHY do these mathematical processes WORK in the first place?
- If  $g(x) = 2x^2$ , solve  $g(x) = 8$
- EXPLAIN the mathematical processes that are going on as you develop your solution → WHY do these mathematical processes WORK in the first place?

17

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### (G) WHY Factor in the First Place

- Now solve  $f(x) + g(x) = 8$
- EXPLAIN the mathematical processes that are going on as you develop your solution → WHY do the previous mathematical processes NOT WORK now?

18

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## (G) WHY Factor in the First Place

- Now solve  $f(x) + g(x) = 8$
- EXPLAIN the mathematical processes that are going on as you develop your solution → WHY do the previous mathematical processes NOTWORK now?
- KEY POINT → When a base function is **isolated**, only then can it “be inverted” in order to facilitate the isolation of the variable.

19

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## (G) WHY Factor in the First Place

- So WHY factor in the first place???
- Think about what we have really done by factoring → we have taken a function which is expressed as sums and/or differences of three “base” functions **AND RE-EXPRESSED** the function as a product of **LINEAR FACTORS**
- So now we have our original function rewritten as LINEAR products → linear functions in which **we can isolate the base function**

20

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## (G) WHY Factor in the First Place

- So as an EXTENSION exercise:
- (a) HOW would you solve  $-x^3 - x^2 + 4x + 4 = 0$ ?
- (b) HOW would you solve  $\sin^2 x - 2\sin x \cos x + \cos^2 x = 0$
- (c) What is the solution set for the equation  $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$

21

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