Lesson 11 – Problem Solving & Applications of Functions

Math 2 Honors - Santowski

Problem 1

- Graph the function y = |x| and select several key points
- Now graph $y = \frac{1}{4} f(2x-2)+1$
- State the new co-ordinates of the key transformed point
- How has the original function been transformed?
- State the domain and range of the new function?
- If g(x) = x - 3 and f(x) = |x|, determine the equation and graph y = f o g(x)

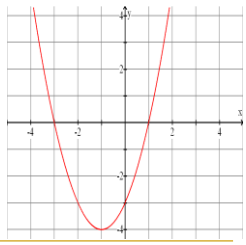
Composing with Linear Functions & Transformations

- Given $f(x) = \sqrt{4-(x)^2}$ and g(x) = x – 3, graph and analyze the function fog(x). Then graph and analyze gof(x)
- Given $f(x) = \sqrt{4-(x)^2}$ and g(x) = 2x, graph and analyze the function fog(x). Then graph and analyze gof(x)
- Given $f(x) = \sqrt{4-(x)^2}$ and g(x) = 2(x – 3), graph and analyze the function fog(x). Then graph and analyze gof(x)
- Given $f(x) = \sqrt{4-(x)^2}$ and g(x) = ax – b, graph and analyze the function fog(x). Then graph and analyze gof(x)

Problem 3

- Consider a graph of the following data:
- 1. State domain and range of *f*
- 2. Evaluate f(-2), f(0),
- 3. Graph the inverse relation
- 4. Is the inverse a function?
- 5. HOW can we make the inverse a function?
- 6. Evaluate $f^{-1}(1)$, $f^{-1}(-2)$
- 7. State the domain and range of $f^{-1}(x)$

- Here is the graph of f(x)



Problem 4

- Determine the equation for, state domain, evaluate y(-2.2) and then graph y(x), given the following four functions that are used to define y(x):

$f(x) = |x|, \quad g(x) = \lfloor x \rfloor, \quad h(x) = 2x-6, \quad t(x) = 2-x^2$

$y(x) = f \circ t(x)$ $\quad y(x) = \dfrac{f(x)}{x}$

$y(x) = t \circ g(x)$ $\quad y(x) = h^{-1} \circ t^{-1}(x)$

$y(x) = h(x) + t(x)$ $\quad y(x) = \dfrac{f(x)}{h(x)}$

$y(x) = f \circ h(x)$ $\quad y(x) = \begin{cases} f(x) & x < 3 \\ t(x) & x \geq 3 \end{cases}$

Problem 5

- Are function operations associative??
- Use algebraic and graphic evidence to support your conclusions if $f(x) = |x|, \quad h(x) = 2x-6, \quad t(x) = 2-x^2$
- (a) is addition? $(f(x)+t(x))+h(x) = ? = f(x)+(t(x)+h(x))$
- (b) is multiplication? $(f(x) \times t(x)) \times h(x) = ? = f(x) \times (t(x) \times h(x))$
- (c) is composition? $(f(x) \circ t(x)) \circ h(x) = ? = f(x) \circ (t(x) \circ h(x))$

S3.1, p222, Q101-2 - Operations

101. Profit Function Suppose that the revenue R, in dollars, from selling x cell phones, in hundreds, is $R(x) = -1.2x^2 + 220x$. The cost C, in dollars, of selling x cell phones is $C(x) = 0.05x^3 - 2x^2 + 65x + 500$.

(a) Find the profit function, $P(x) = R(x) - C(x)$.

(b) Find the profit if $x = 15$ hundred cell phones are sold.

(c) Interpret $P(15)$.

102. Profit Function Suppose that the revenue R, in dollars, from selling x clocks is $R(x) = 30x$. The cost C, in dollars, of selling x clocks is $C(x) = 0.1x^2 + 7x + 400$.

(a) Find the profit function, $P(x) = R(x) - C(x)$.

(b) Find the profit if $x = 30$ clocks are sold.

(c) Interpret $P(30)$.

S3.1, p222, Q103 - Operations

103. Some functions f have the property that $f(a + b) = f(a) + f(b)$ for all real numbers a and b. Which of the following functions have this property?

(a) $h(x) = 2x$ (b) $g(x) = x^2$

(c) $F(x) = 5x - 2$ (d) $G(x) = \frac{1}{x}$

S3.1, p222, Q97-99 - Operations

97. Economics The **participation rate** is the number of people in the labor force divided by the civilian population (excludes military). Let $L(x)$ represent the size of the labor force in year x and $P(x)$ represent the civilian population in year x. Determine a function that represents the participation rate R as a function of x.

98. Crimes Suppose that $V(x)$ represents the number of violent crimes committed in year x and $P(x)$ represents the number of property crimes committed in year x. Determine a function T that represents the combined total of violent crimes and property crimes in year x.

99. Health Care Suppose that $P(x)$ represents the percentage of income spent on health care in year x and $I(x)$ represents income in year x. Determine a function H that represents total health care expenditures in year x.

S3.2, p227, Q6-8 – Intro to Functions

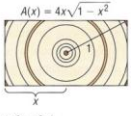
6. *True or False* A function can have more than one y-intercept.

7. *True or False* The graph of a function $y = f(x)$ always crosses the y-axis.

8. *True or False* The y-intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.

S3.2, p227, Q32 – Intro to Fcns

32. Cross-sectional Area The cross-sectional area of a beam cut from a log with radius 1 foot is given by the function $A(x) = 4x\sqrt{1 - x^2}$, where x represents the length, in feet, of half the base of the beam. See the figure.



(a) Find the domain of A.

(b) Use a graphing utility to graph the function $A = A(x)$.

(c) Create a TABLE with TblStart = 0 and ΔTbl = 0.1 for $0 \le x \le 1$. Which value of x maximizes the cross-sectional area? What should be the length of the base of the beam to maximize the cross-sectional area?

S3.2, p227, Q33 – Intro to Fcns

33. Cost of Trans-Atlantic Travel A Boeing 747 crosses the Atlantic Ocean (3000 miles) with an airspeed of 500 miles per hour. The cost C (in dollars) per passenger is given by

$$C(x) = 100 + \frac{x}{10} + \frac{36{,}000}{x}$$

where x is the ground speed (airspeed ± wind).

(a) Use a graphing utility to graph the function $C = C(x)$.

(b) Create a TABLE with TblStart = 0 and ΔTbl = 50.

(c) To the nearest 50 miles per hour, what ground speed minimizes the cost per passenger?

S3.2, p227, Q34 – Intro to Fcns

34. Effect of Elevation on Weight If an object weighs m pounds at sea level, then its weight W (in pounds) at a height of h miles above sea level is given approximately by

$$W(h) = m\left(\frac{4000}{4000 + h}\right)^2$$

(a) If Amy weighs 120 pounds at sea level, how much will she weigh on Pike's Peak, which is 14,110 feet above sea level?

(b) Use a graphing utility to graph the function $W = W(h)$. Use $m = 120$ pounds.

(c) Create a Table with TblStart $= 0$ and ΔTbl $= 0.5$ to see how the weight W varies as h changes from 0 to 5 miles.

(d) At what height will Amy weigh 119.95 pounds?

(e) Does your answer to part (d) seem reasonable? Explain.

S6.2, p422, Q93 - Inverses

93. Gravity on Earth If a rock falls from a height of 100 meters on Earth, the height H (in meters) after t seconds is approximately

$$H(t) = 100 - 4.9t^2$$

(a) In general, quadratic functions are not one-to-one. However, the function $H(t)$ is one-to-one. Why?

(b) Find the inverse of H and verify your result.

(c) How long will it take a rock to fall 80 meters?

S6.2, p422, Q94 - Inverses

94. Period of a Pendulum The period T (in seconds) of a simple pendulum as a function of its length l (in feet) is given by

$$T(l) = 2\pi\sqrt{\frac{l}{32.2}}$$

(a) Express the length l as a function of the period T.

(b) How long is a pendulum whose period is 3 seconds?

S6.2, p422, Q95 - Inverses

95. Given

$$f(x) = \frac{ax + b}{cx + d}$$

find $f^{-1}(x)$. If $c \neq 0$, under what conditions on a, b, c, and d is $f = f^{-1}$?

S3.4, p250, Q47 – Special Functions

47. Cell Phone Service Sprint PCS offers a monthly cellular phone plan for $35. It includes 300 anytime minutes and charges $0.40 per minute for additional minutes. The following function is used to compute the monthly cost for a subscriber:

$$C(x) = \begin{cases} 35 & \text{if } 0 < x \leq 300 \\ 0.40x - 85 & \text{if } x > 300 \end{cases}$$

where x is the number of anytime minutes used. Compute the monthly cost of the cellular phone for use of the following anytime minutes:

(a) 200 (b) 365 (c) 301

Source: Sprint PCS

S3.4, p250, Q48 – Special Functions

48. Parking at O'Hare International Airport The short-term (no more than 24 hours) parking fee F (in dollars) for parking x hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 3 & \text{if } 0 < x \leq 3 \\ 5 \text{ int}(x + 1) + 1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \leq x \leq 24 \end{cases}$$

Determine the fee for parking in the short-term parking garage for

(a) 2 hours

(b) 7 hours

(c) 15 hours

(d) 8 hours and 24 minutes

Source: O'Hare International Airport

S3.4, p250, Q49 – Special Functions

49. Cost of Natural Gas In May 2006, Peoples Energy had the following rate schedule for natural gas usage in single-family residences:

Monthly service charge	\$9.45
Per therm service charge	
1st 50 therms	\$0.36375/therm
Over 50 therms	\$0.11445/therm
Gas charge	\$0.7958/therm

(a) What is the charge for using 50 therms in a month?
(b) What is the charge for using 500 therms in a month?
(c) Construct a function that relates the monthly charge C for x therms of gas.
(d) Graph this function.

Source: Peoples Energy, Chicago, Illinois, 2006

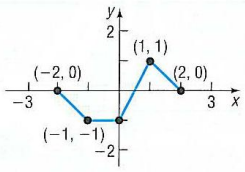
S3.5, p263, Q70,72 - Transformations

70. Suppose that the x-intercepts of the graph of $y = f(x)$ are -8 and 1.
(a) What are the x-intercepts of the graph of $y = f(x + 4)$?
(b) What are the x-intercepts of the graph of $y = f(x - 3)$?
(c) What are the x-intercepts of the graph of $y = 2f(x)$?
(d) What are the x-intercepts of the graph of $y = f(-x)$?

72. Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.
(a) Over what interval is the graph of $y = f(x + 2)$ decreasing?
(b) Over what interval is the graph of $y = f(x - 5)$ decreasing?
(c) What can be said about the graph of $y = -f(x)$?
(d) What can be said about the graph of $y = f(-x)$?

S3.5, p263, Q74 - Transformations

74. The graph of a function f is illustrated in the figure.
(a) Draw the graph of $y = |f(x)|$.
(b) Draw the graph of $y = f(|x|)$.



S3.5, p263, Q88 - Transformations

88. Digital Music Revenues The total projected worldwide digital music revenues R, in millions of dollars, for the years 2005 through 2010 can be estimated by the function

$$R(x) = 170.7x^2 + 1373x + 1080$$

where x is the number of years after 2005.
(a) Find $R(0)$, $R(3)$, and $R(5)$ and explain what each value represents.
(b) Find $r = R(x - 5)$.
(c) Find $r(5)$, $r(8)$, and $r(10)$ and explain what each value represents.
(d) In the model r, what does x represent?
(e) Would there be an advantage in using the model r when estimating the projected revenues for a given year instead of the model R?

Source: eMarketer.com, May 2006

S3.5, p263, Q90 - Transformations

90. Period of a Pendulum The period T (in seconds) of a simple pendulum is a function of its length l (in feet) defined by the equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $g \approx 32.2$ feet per second per second is the acceleration of gravity.
(a) Use a graphing utility to graph the function $T = T(l)$.
(b) Now graph the functions $T = T(l + 1)$, $T = T(l + 2)$, and $T = T(l + 3)$.
(c) Discuss how adding to the length l changes the period T.
(d) Now graph the functions $T = T(2l)$, $T = T(3l)$, and $T = T(4l)$.
(e) Discuss how multiplying the length l by factors of 2, 3, and 4 changes the period T.

S3.5, p263, Q91 - Transformations

91. Cigar Company Profits The daily profits of a cigar company from selling x cigars are given by

$$p(x) = -0.05x^2 + 100x - 2000$$

The government wishes to impose a tax on cigars (sometimes called a *sin tax*) that gives the company the option of either paying a flat tax of \$10,000 per day or a tax of 10% on profits. As chief financial officer (CFO) of the company, you need to decide which tax is the better option for the company.
(a) On the same screen, graph $Y_1 = p(x) - 10{,}000$ and $Y_2 = (1 - 0.10)p(x)$.
(b) Based on the graph, which option would you select? Why?
(c) Using the terminology learned in this section, describe each graph in terms of the graph of $p(x)$.
(d) Suppose that the government offered the options of a flat tax of \$4800 or a tax of 10% on profits. Which would you select? Why?