

Lesson 10 – Transformations of Functions

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Lesson Objectives

- ▶ Review and graph piecewise functions, step functions, and absolute-value functions
- ▶ Identify and graph transformations of functions
- ▶ BIG PICTURE → knowing how to transform BASE functions gives you the tools necessary to analyze and apply a much wider range of functions into real world modeling scenarios

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Skills Preview

- ▶ Describe the transformations of $y = f(x)$ implied in the equation:
- ▶ $y = a f(b(x + c)) + d$
- ▶ a and d affect →
- ▶ b and c affect →

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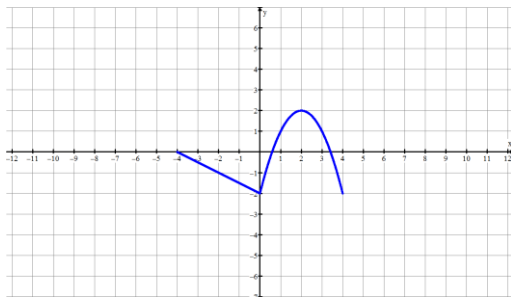
Skills Preview

- ▶ Graph the function: $f(x) = \begin{cases} \frac{1}{2}x - 2 & \text{if } -4 \leq x \leq 0 \\ -(x-2)^2 + 2 & \text{if } 0 < x \leq 4 \end{cases}$
- ▶ Evaluate the key points at $f(-4)$, $f(0)$, $f(2)$, $f(4)$
- ▶ Graph $y = -\frac{1}{2}f(2x+8) - 1$

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Skills Preview



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(A) Transformations $y = af(x)$

- ▶ Graph the function $f(x) = \sqrt{4-x^2}$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window
- ▶ Now graph $y = 2f(x)$, $y = 3f(x)$, $y = 4f(x)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function
- ▶ Determine the equation of $y = f^{-1}(x)$

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(A) Transformations $y = af(x)$

- ▶ Graph the function $f(x) = \sqrt{4-x^2}$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = \frac{1}{2}f(x), y = \frac{1}{3}f(x), y = \frac{1}{4}f(x)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function

- ▶ Determine $f \circ f^{-1}(x)$

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(A) Transformations $y = af(x)$

- ▶ Graph the function $f(x) = \sqrt{4-x^2}$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = -f(x)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function

- ▶ Graph $y = \lfloor f(x) \rfloor$

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(B) Transformations $y = f(bx)$

- ▶ Graph the function $f(x) = \sqrt{4-(x)^2}$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(2x), y = f(3x), y = f(4x)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function

- ▶ Graph $y = \lfloor f^{-1}(x) \rfloor$

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(B) Transformations $y = f(bx)$

- ▶ Graph the function $f(x) = \sqrt{4-(x)^2}$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(\frac{1}{2}x), y = f(\frac{1}{3}x), y = f(\frac{1}{4}x)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function

- ▶ Graph $y = \frac{1}{f(x)}$

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(c) Transformations $y = f(x + c)$

- ▶ Graph the function $y = |x|$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(x + 2), y = f(x + 3), y = f(x + 4)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function?

- ▶ If $g(x) = x + 2$ and $f(x) = |x|$, determine the equation and graph $y = f \circ g(x)$

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(c) Transformations $y = f(x + c)$

- ▶ Graph the function $y = |x|$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(x - 2), y = f(x - 3), y = f(x - 4)$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function?

- ▶ If $g(x) = x - 3$ and $f(x) = |x|$, determine the equation and graph $y = f \circ g(x)$

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(d) Transformations $y = f(x) + d$

- ▶ Graph the function $f(x) = \lfloor x \rfloor$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(x) + 2$, $y = f(x) + 3$, $y = f(x) + 4$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function?

- ▶ Determine the equation for $y = f^{-1}(x)$

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(d) Transformations $y = f(x) + d$

- ▶ Graph the function $f(x) = \lfloor x \rfloor$
- ▶ State the domain and range of $f(x)$
- ▶ View in a squared window

- ▶ Now graph $y = f(x) - 2$, $y = f(x) - 3$, $y = f(x) - 4$
- ▶ State the new co-ordinates of the key transformed point
- ▶ How has the original function been transformed?
- ▶ State the domain and range of the new function?

- ▶ Determine the equation for $y = f^{-1}(x)$

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Composing with Linear Functions & Transformations

- ▶ Given $f(x) = \sqrt{4 - (x)^2}$ and $g(x) = x - 3$, graph and analyze the function $f \circ g(x)$. Then graph and analyze $g \circ f(x)$
- ▶ Given $f(x) = \sqrt{4 - (x)^2}$ and $g(x) = 2x$, graph and analyze the function $f \circ g(x)$. Then graph and analyze $g \circ f(x)$
- ▶ Given $f(x) = \sqrt{4 - (x)^2}$ and $g(x) = 2(x - 3)$, graph and analyze the function $f \circ g(x)$. Then graph and analyze $g \circ f(x)$
- ▶ Given $f(x) = \sqrt{4 - (x)^2}$ and $g(x) = ax - b$, graph and analyze the function $f \circ g(x)$. Then graph and analyze $g \circ f(x)$

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WHY do we need Transformations?

- ▶ On the next slide, you will find a picture of my friend who visited Arches National Park a couple of years ago. She is 5 foot 6 inches tall. Determine an equation you can use to model the arch under which she is standing

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WHY do we need Transformations?



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WHY do we need Transformations?

- ▶ On the next slide, you will find a picture of Sydney Harbour Bridge. The bottom "arch" is shaped as a parabola. You will need to research some data about the bridge so that you can determine an equation you can use to model the bridge

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WHY do we need Transformations?



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(f) Homework

- ▶ p. 139 # 20,21,22,25,35,37,39,41,52,53,57,58, 62-65
- ▶ Sullivan, Sec 3.8, p261-4, Q7-18 (graph matching), 74

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Links

- ▶ Video Link#1 -
<http://www.youtube.com/watch?v=3Q5Sy034fok>
- ▶ Video Link #2 -
<http://www.youtube.com/watch?v=LAH8A4CaDYM&feature=related>
- ▶ Link from PurpleMath →
<http://www.purplemath.com/modules/fcntrans.htm>

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