Algebra II: Unit 6 Test /75 Name: _____

Part B – No Calculator: Show all work for full credit in the space provided.

1. Perform long division, and state the quotient and remainder: $\frac{3x^2+4x-5}{x-2}$

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2. Perform synthetic division, and state the quotient and remainder: $\frac{x^3-2x^2+3}{x+1}$

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3. Write the name of the theorem underneath its description.

A) The remainder of $P(x)$ divided by $(x-k)$ is $P(k)$.

B) If a polynomial with rational coefficients has an irrational root, then its conjugate is also a root.

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C) A polynomial of degree *n* has *n* (complex or real) roots.

D) If a polynomial with rational coefficients has an imaginary root, then its conjugate is also a root.

E) The possible rational roots of a polynomial come from the ratios of constant factors over leading coefficient factors.

F) A polynomial $P(x)$ has a factor of $(x-k)$ if and only if its remainder is zero: $P(k)=0$.

4. Fully factor the following polynomials. Use *i* where necessary.

   a) $x^4 - 81$ b) $8x^3 + 27$ c) $12x^2 - 8x - 15$ d) $2x^3 - x^2 - 6x$

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5. Solve for all real and complex roots:

   a) $x^2 = -25$ b) $3x^2 - 9 = 0$ c) $x^4 - 1 = 0$

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6. Given the polynomial $P(x) = x^6 + 3x^5 - 2x^4 + 10x^3 - x^2 - 7x - 4$.

   a) Use the Remainder Theorem to determine the remainder when $P(x)$ is divided by $(x-1)$.

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   b) Is $(x-1)$ a factor of $P(x)$? Why or why not?

7. Write the factored form of the equation of a 5^{th} degree polynomial given the following roots:

   a) $x = 2$ (multiplicity 3) and $x = -3$ (multiplicity 2)

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   b) $x = 0$, $x = 3 - 2i$ and $x = -1 + \sqrt{2}$

8. Complete the table for the possible number and kinds of roots for ANY CUBIC POLYNOMIAL. There may not be 6 different cases.

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	Number of Real Roots		Number of Complex Roots	Total Number of Roots
	Rational	Irrational		
Case 1:				
Case 2:				
Case 3:				
Case 4:				
Case 5:				
Case 6:				

9. Given $x^3 - 4x^2 + x + 6 = 0$.

a) Is this a polynomial function or equation? Explain. (1 mark)

b) What is the minimum number of rational roots? Explain. (2 marks)

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c) List all of the POSSIBLE rational roots of $x^3 - 4x^2 + x + 6$ (1 mark)

d) Is $x = 1$ an ACTUAL root of $x^3 - 4x^2 + x + 6 = 0$? (2 marks)

e) Is $x = -1$ an ACTUAL root of $x^3 - 4x^2 + x + 6 = 0$? (2 marks)

f) What is one linear factor of $x^3 - 4x^2 + x + 6$? (1 mark)

g) Divide $x^3 - 4x^2 + x + 6$ by the linear factor found in (f) and write $x^3 - 4x^2 + x + 6$ as the product of a linear polynomial and a quadratic polynomial. (2 marks)

h) Fully factor $x^3 - 4x^2 + x + 6$. (3 marks)

i) Solve $x^3 - 4x^2 + x + 6 = 0$ for all real and complex roots. (3 marks)

j) Verify all of your solutions to $x^3 - 4x^2 + x + 6 = 0$. (1 mark)

BONUS: Solve $x^5 - x^4 - x^3 + x^2 - 2x + 2 = 0$ for all real roots (including rational and irrational) and complex roots.

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