If $|c| \le \sqrt{a^2 + b^2}$, then all the solutions of equation (5) are

$$\theta + \phi = \sin^{-1}\frac{c}{\sqrt{a^2 + b^2}} + 2k\pi \quad \text{or} \quad \theta + \phi = \pi - \sin^{-1}\frac{c}{\sqrt{a^2 + b^2}} + 2k\pi$$

Because the angle ϕ is determined by equations (4), these provide the solutions to equation (2).

Now Work PROBLEM 41

4 Solve Trigonometric Equations Using a Graphing Utility

The techniques introduced in this section apply only to certain types of trigonometric equations. Solutions for other types are usually studied in calculus, using numerical methods. In the next example, we show how a graphing utility may be used to obtain solutions.

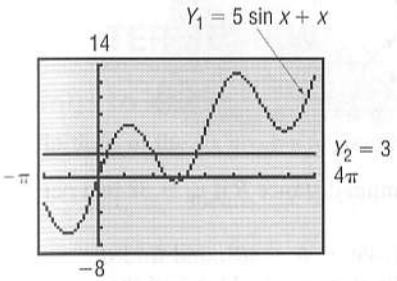
EXAMPLE 8 Solving Trigonometric Equations Using a Graphing Utility

Solve: $5 \sin x + x = 3$

Express the solution(s) rounded to two decimal places.

Solution This type of trigonometric equation cannot be solved by previous methods. A graphing utility, though, can be used here. The solution(s) of this equation is the same as the points of intersection of the graphs of $Y_1 = 5 \sin x + x$ and $Y_2 = 3$. See Figure 32.

Figure 32



There are three points of intersection; the x-coordinates are the solutions that we seek. Using INTERSECT, we find

$$x = 0.52, \qquad x = 3.18, \qquad x = 5.71$$

The solution set is $\{0.52, 3.18, 5.71\}$.

Now Work PROBLEM 53

8.8 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Find the real solutions of $4x^2 - x - 5 = 0$. (pp. 98–99)
2. Find the real solutions of $x^2 - x - 1 = 0$. (pp. 102–109)
3. Find the real solutions of $(2x - 1)^2 - 3(2x - 1) - 4 = 0$. (pp. 119–121)
4. Use a graphing utility to solve $5x^3 - 2 = x - x^2$. Round answers to two decimal places. (pp. A8–A10)

Skill Building

In Problems 5–46, solve each equation on the interval $0 \le \theta < 2\pi$.

5. $2\cos^2\theta + \cos\theta = 0$
6. $\sin^2\theta - 1 = 0$
7. $2\sin^2\theta - \sin\theta - 1 = 0$
8. $2\cos^2\theta + \cos\theta - 1 = 0$
9. $(\tan\theta - 1)(\sec\theta - 1) = 0$
10. $(\cot\theta + 1)\left(\csc\theta - \frac{1}{2}\right) = 0$
11. $\sin^2\theta - \cos^2\theta = 1 + \cos\theta$
12. $\cos^2\theta - \sin^2\theta + \sin\theta = 0$
13. $\sin^2\theta = 6(\cos\theta + 1)$
14. $2\sin^2\theta = 3(1 - \cos\theta)$
15. $\cos(2\theta) + 6\sin^2\theta = 4$
16. $\cos(2\theta) = 2 - 2\sin^2\theta$
17. $\cos\theta = \sin\theta$
18. $\cos\theta + \sin\theta = 0$
19. $\tan\theta = 2\sin\theta$
20. $\sin(2\theta) = \cos\theta$
21. $\sin\theta = \csc\theta$
22. $\tan\theta = \cot\theta$
23. $\cos(2\theta) = \cos\theta$
24. $\sin(2\theta)\sin\theta = \cos\theta$
25. $\sin(2\theta) + \sin(4\theta) = 0$
26. $\cos(2\theta) + \cos(4\theta) = 0$
27. $\cos(4\theta) - \cos(6\theta) = 0$
28. $\sin(4\theta) - \sin(6\theta) = 0$

29. $1 + \sin\theta = 2\cos^2\theta$ **30.** $\sin^2\theta = 2\cos\theta + 2$ **31.** $2\sin^2\theta - 5\sin\theta + 3 = 0$

32. $2\cos^2\theta - 7\cos\theta - 4 = 0$ **33.** $3(1 - \cos\theta) = \sin^2\theta$ **34.** $4(1 + \sin\theta) = \cos^2\theta$

35. $\tan^2\theta = \frac{3}{2}\sec\theta$ **36.** $\csc^2\theta = \cot\theta + 1$ **37.** $3 - \sin\theta = \cos(2\theta)$

38. $\cos(2\theta) + 5\cos\theta + 3 = 0$ **39.** $\sec^2\theta + \tan\theta = 0$ **40.** $\sec\theta = \tan\theta + \cot\theta$

41. $\sin\theta - \sqrt{3}\cos\theta = 1$ **42.** $\sqrt{3}\sin\theta + \cos\theta = 1$ **43.** $\tan(2\theta) + 2\sin\theta = 0$

44. $\tan(2\theta) + 2\cos\theta = 0$ **45.** $\sin\theta + \cos\theta = \sqrt{2}$ **46.** $\sin\theta + \cos\theta = -\sqrt{2}$

In Problems 47–52, find the real zeros of each trigonometric function on the interval $0 \le x < 2\pi$.

47. $f(x) = 4\cos^2 x - 1$ **48.** $f(x) = 4\sin^2 x - 3$ **49.** $f(x) = \sin(2x) - \sin x$

50. $f(x) = \cos(2x) + \cos x$ **51.** $f(x) = \sin^2 x + 2\cos x + 2$ **52.** $f(x) = \cos(2x) + \sin^2 x$

In Problems 53–64, use a graphing utility to solve each equation. Express the solution(s) rounded to two decimal places.

53. $x + 5\cos x = 0$ **54.** $x - 4\sin x = 0$ **55.** $22x - 17\sin x = 3$

56. $19x + 8\cos x = 2$ **57.** $\sin x + \cos x = x$ **58.** $\sin x - \cos x = x$

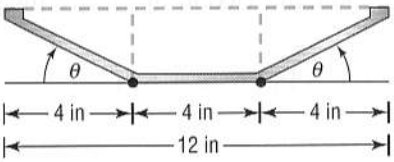
59. $x^2 - 2\cos x = 0$ **60.** $x^2 + 3\sin x = 0$ **61.** $x^2 - 2\sin(2x) = 3x$

62. $x^2 = x + 3\cos(2x)$ **63.** $6\sin x - e^x = 2, \quad x > 0$ **64.** $4\cos(3x) - e^x = 1, \quad x > 0$

Applications and Extensions

65. Constructing a Rain Gutter A rain gutter is to be constructed of aluminum sheets 12 inches wide. After marking off a length of 4 inches from each edge, this length is bent up at an angle θ. See the illustration. The area A of the opening as a function of θ is given by

$$A(\theta) = 16\sin\theta(\cos\theta + 1) \quad 0° < \theta < 90°$$



(a) In calculus, you will be asked to find the angle θ that maximizes A by solving the equation

$$\cos(2\theta) + \cos\theta = 0, \quad 0° < \theta < 90°$$

Solve this equation for θ by using the Double-angle Formula.

(b) Solve the equation in part (a) for θ by writing the sum of the two cosines as a product.

(c) What is the maximum area A of the opening?

(d) Graph $A = A(\theta), 0° \le \theta \le 90°$, and find the angle θ that maximizes the area A. Also find the maximum area. Compare the results to the answers found earlier.

66. Projectile Motion An object is propelled upward at an angle $\theta, 45° < \theta < 90°$, to the horizontal with an initial velocity of v_0 feet per second from the base of an inclined plane that makes an angle of 45° with the horizontal. See the illustration. If air resistance is ignored, the distance R that the object travels up the inclined plane is given by

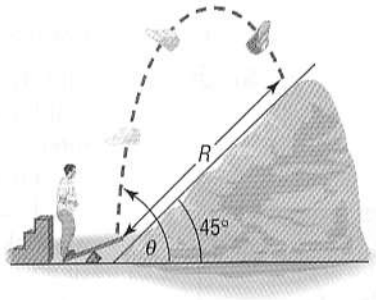
$$R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$$

(a) In calculus, you will be asked to find the angle θ that maximizes R by solving the equation

$$\sin(2\theta) + \cos(2\theta) = 0$$

Solve this equation for θ using the method of Example 7.

(b) Solve this equation for θ by dividing each side by $\cos(2\theta)$.



(c) What is the maximum distance R if $v_0 = 32$ feet per second?

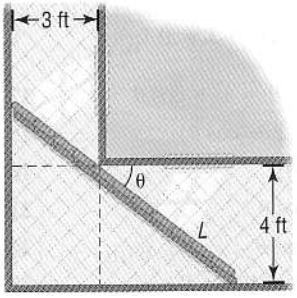
(d) Graph $R = R(\theta), 45° \le \theta \le 90°$, and find the angle θ that maximizes the distance R. Also find the maximum distance. Use $v_0 = 32$ feet per second. Compare the results with the answers found earlier.

67. Heat Transfer In the study of heat transfer, the equation $x + \tan x = 0$ occurs. Graph $Y_1 = -x$ and $Y_2 = \tan x$ for $x \ge 0$. Conclude that there are an infinite number of points of intersection of these two graphs. Now find the first two positive solutions of $x + \tan x = 0$ rounded to two decimal places.

68. Carrying a Ladder Around a Corner Two hallways, one of width 3 feet, the other of width 4 feet, meet at a right angle. See the illustration.

(a) Show that the length L as a function of θ is

$$L(\theta) = 4\csc\theta + 3\sec\theta$$



43.
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\sin\alpha\cos\beta$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

45. $2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2} = 2\cdot\dfrac{1}{2}\left[\cos\left(\dfrac{\alpha+\beta}{2}+\dfrac{\alpha-\beta}{2}\right)+\cos\left(\dfrac{\alpha+\beta}{2}-\dfrac{\alpha-\beta}{2}\right)\right] = \cos\dfrac{2\alpha}{2}+\cos\dfrac{2\beta}{2} = \cos\alpha+\cos\beta$

8.7 Assess Your Understanding *(page 653)*

3. $\frac{\pi}{6}, \frac{5\pi}{6}$ **4.** $\left\{\theta \middle| \theta = \frac{\pi}{6} + 2\pi k, \theta = \frac{5\pi}{6} + 2\pi k, k \text{ any integer}\right\}$ **5.** F **6.** F **7.** $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **9.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **11.** $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

13. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **15.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **17.** $\left\{\frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}\right\}$ **19.** $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **21.** $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **23.** $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

25. $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ **27.** $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **29.** $\left\{\frac{11\pi}{6}\right\}$ **31.** $\left\{\theta \middle| \theta = \frac{\pi}{6} + 2k\pi, \theta = \frac{5\pi}{6} + 2k\pi\right\}$; $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

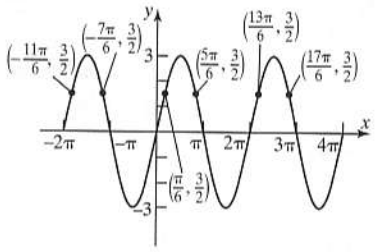
33. $\left\{\theta \middle| \theta = \frac{5\pi}{6} + k\pi\right\}$; $\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}, \frac{29\pi}{6}, \frac{35\pi}{6}$ **35.** $\left\{\theta \middle| \theta = \frac{\pi}{2} + 2k\pi, \theta = \frac{3\pi}{2} + 2k\pi\right\}$; $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

37. $\left\{\theta \middle| \theta = \frac{\pi}{3} + k\pi, \theta = \frac{2\pi}{3} + k\pi\right\}$; $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$ **39.** $\left\{\theta \middle| \theta = \frac{8\pi}{3} + 4k\pi, \theta = \frac{10\pi}{3} + 4k\pi\right\}$; $\frac{8\pi}{3}, \frac{10\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}, \frac{32\pi}{3}, \frac{34\pi}{3}$

41. $\{0.41, 2.73\}$ **43.** $\{1.37, 4.51\}$ **45.** $\{2.69, 3.59\}$ **47.** $\{1.82, 4.46\}$ **49.** $\{2.08, 5.22\}$ **51.** $\{0.73, 2.41\}$ **53.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

55. **(a)** $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

(b)

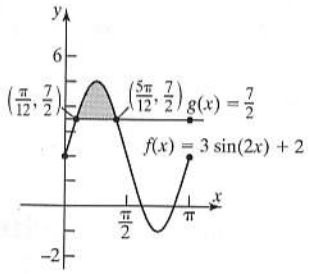


(c) $\left\{-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}$

(d) $\left\{x \middle| -\frac{11\pi}{6} < x < -\frac{7\pi}{6} \text{ or } \frac{\pi}{6} < x < \frac{5\pi}{6} \text{ or } \frac{13\pi}{6} < x < \frac{17\pi}{6}\right\}$

57. **(a)** $\left\{x \middle| x = -\frac{\pi}{4} + k\pi\right\}$ **(b)** $-\frac{\pi}{2} < x < -\frac{\pi}{4}$ or $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$

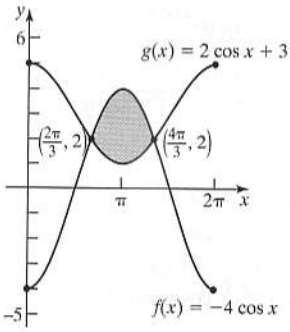
59. **(a), (d)**



(b) $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$

(c) $\left\{x \middle| \frac{\pi}{12} < x < \frac{5\pi}{12}\right\}$ or $\left(\frac{\pi}{12}, \frac{5\pi}{12}\right)$

61. **(a), (d)**



(b) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

(c) $\left\{x \middle| \frac{2\pi}{3} < x < \frac{4\pi}{3}\right\}$ or $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

63. **(a)** 10 sec; 30 sec **(b)** 20 sec; 60 sec **(c)** $10 < x < 30$ or $(10, 30)$

65. **(a)** 150 mi **(b)** 6.06, 8.44, 15.72, 18.11 min **(c)** Before 6.06 min, between 8.44 and 15.72 min, and after 18.11 min **(d)** No

67. 28.90° **69.** Yes; it varies from 1.25 to 1.34.

71. 1.47

73. If θ is the original angle of incidence and ϕ is the angle of refraction, then $\dfrac{\sin\theta}{\sin\phi} = n_2$.

The angle of incidence of the emerging beam is also ϕ, and the index of refraction is $\dfrac{1}{n_2}$.

Thus, θ is the angle of refraction of the emerging beam.

8.8 Assess Your Understanding *(page 661)*

5. $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ **7.** $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **9.** $\left\{0, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$ **11.** $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ **13.** $\{\pi\}$ **15.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **17.** $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$ **19.** $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

21. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ **23.** $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ **25.** $\left\{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$ **27.** $\left\{0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}\right\}$ **29.** $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ **31.** $\left\{\frac{\pi}{2}\right\}$

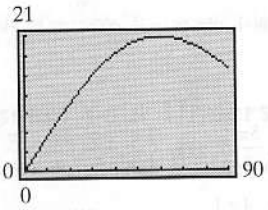
33. $\{0\}$ **35.** $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **37.** No real solution **39.** No real solution **41.** $\left\{\frac{\pi}{2}, \frac{7\pi}{6}\right\}$ **43.** $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$ **45.** $\left\{\frac{\pi}{4}\right\}$ **47.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

49. $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **51.** π **53.** $-1.31, 1.98, 3.84$ **55.** 0.52 **57.** 1.26 **59.** $-1.02, 1.02$ **61.** $0, 2.15$ **63.** $0.76, 1.35$

65. **(a)** $60°$ **(b)** $60°$

(c) $A(60°) = 12\sqrt{3}$ in.^{2}

(d)



$\theta_{max} = 60°$

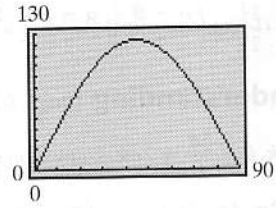
Maximum area $= 20.78$ in.^{2}

67. 2.03, 4.91

69. **(a)** $30°, 60°$

(b) 123.6 m

(c)



Review Exercises *(page 664)*

1. $\frac{\pi}{2}$ **3.** $\frac{\pi}{4}$ **5.** $\frac{5\pi}{6}$ **7.** $\frac{\pi}{4}$ **9.** $\frac{3\pi}{8}$ **11.** $-\frac{\pi}{3}$ **13.** $\frac{\pi}{7}$ **15.** 0.9 **17.** -0.3 **19.** Not defined **21.** $-\frac{\pi}{6}$ **23.** $-\frac{\pi}{4}$ **25.** $-\sqrt{3}$ **27.** $\frac{2\sqrt{3}}{3}$

29. $\frac{4}{5}$ **31.** $-\frac{4}{3}$ **33.** $f^{-1}(x) = \frac{1}{3}\sin^{-1}\left(\frac{x}{2}\right)$; Domain of f: $(-\infty, \infty)$; Domain of f^{-1}: $[-2, 2]$

35. $f^{-1}(x) = \cos^{-1}(3 - x)$; Domain of f: $(-\infty, \infty)$; Domain of f^{-1}: $[2, 4]$ **37.** $\sqrt{1 - u^2}$ **39.** $\frac{1}{u}$ **41.** $\tan\theta\cot\theta - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta$

43. $\sin^2\theta(1 + \cot^2\theta) = \sin^2\theta\csc^2\theta = 1$ **45.** $5\cos^2\theta + 3\sin^2\theta = 2\cos^2\theta + 3(\cos^2\theta + \sin^2\theta) = 3 + 2\cos^2\theta$

47. $\dfrac{1 - \cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1 - \cos\theta} = \dfrac{(1 - \cos\theta)^2 + \sin^2\theta}{\sin\theta(1 - \cos\theta)} = \dfrac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1 - \cos\theta)} = \dfrac{2(1 - \cos\theta)}{\sin\theta(1 - \cos\theta)} = 2\csc\theta$

49. $\dfrac{\cos\theta}{\cos\theta - \sin\theta} = \dfrac{\dfrac{\cos\theta}{\cos\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}} = \dfrac{1}{1 - \dfrac{\sin\theta}{\cos\theta}} = \dfrac{1}{1 - \tan\theta}$

51. $\dfrac{\csc\theta}{1 + \csc\theta} = \dfrac{\dfrac{1}{\sin\theta}}{1 + \dfrac{1}{\sin\theta}} = \dfrac{1}{1 + \sin\theta} = \dfrac{1}{1 + \sin\theta}\cdot\dfrac{1 - \sin\theta}{1 - \sin\theta} = \dfrac{1 - \sin\theta}{1 - \sin^2\theta} = \dfrac{1 - \sin\theta}{\cos^2\theta}$

53. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1 - \sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta\cdot\dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$

55. $\dfrac{1 - \sin\theta}{\sec\theta} = \cos\theta(1 - \sin\theta)\cdot\dfrac{1 + \sin\theta}{1 + \sin\theta} = \dfrac{\cos\theta(1 - \sin^2\theta)}{1 + \sin\theta} = \dfrac{\cos^3\theta}{1 + \sin\theta}$

57. $\cot\theta - \tan\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$

59. $\dfrac{\cos(\alpha + \beta)}{\cos\alpha\sin\beta} = \dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \dfrac{\cos\alpha\cos\beta}{\cos\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \cot\beta - \tan\alpha$

61. $\dfrac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$

63. $(1 + \cos\theta)\left(\tan\dfrac{\theta}{2}\right) = (1 + \cos\theta)\cdot\dfrac{\sin\theta}{1 + \cos\theta} = \sin\theta$

65. $2\cot\theta\cot 2\theta = 2\left(\dfrac{\cos\theta}{\sin\theta}\right)\left(\dfrac{\cos 2\theta}{\sin 2\theta}\right) = \dfrac{2\cos\theta(\cos^2\theta - \sin^2\theta)}{2\sin^2\theta\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} = \cot^2\theta - 1$

67. $1 - 8\sin^2\theta\cos^2\theta = 1 - 2(2\sin\theta\cos\theta)^2 = 1 - 2\sin^2(2\theta) = \cos(4\theta)$ **69.** $\dfrac{\sin(2\theta) + \sin(4\theta)}{\cos(2\theta) + \cos(4\theta)} = \dfrac{2\sin(3\theta)\cos(-\theta)}{2\cos(3\theta)\cos(-\theta)} = \tan(3\theta)$

71. $\dfrac{\cos(2\theta) - \cos(4\theta)}{\cos(2\theta) + \cos(4\theta)} - \tan\theta\tan(3\theta) = \dfrac{-2\sin(3\theta)\sin(-\theta)}{2\cos(3\theta)\cos(-\theta)} - \tan\theta\tan(3\theta) = \tan(3\theta)\tan\theta - \tan\theta\tan(3\theta) = 0$

73. $\frac{1}{4}(\sqrt{6} - \sqrt{2})$ **75.** $\frac{1}{4}(\sqrt{6} - \sqrt{2})$ **77.** $\frac{1}{2}$ **79.** $\sqrt{2} - 1$ **81.** **(a)** $-\frac{33}{65}$ **(b)** $-\frac{56}{65}$ **(c)** $-\frac{63}{65}$ **(d)** $\frac{33}{56}$ **(e)** $\frac{24}{25}$ **(f)** $\frac{119}{169}$ **(g)** $\frac{5\sqrt{26}}{26}$ **(h)** $\frac{2\sqrt{5}}{5}$ **83.** **(a)** $-\frac{16}{65}$ **(b)** $-\frac{63}{65}$ **(c)** $-\frac{56}{65}$ **(d)** $\frac{16}{63}$ **(e)** $\frac{24}{25}$ **(f)** $\frac{119}{169}$ **(g)** $\frac{\sqrt{26}}{26}$ **(h)** $-\frac{\sqrt{10}}{10}$ **85.** **(a)** $-\frac{63}{65}$ **(b)** $\frac{16}{65}$ **(c)** $\frac{33}{65}$ **(d)** $-\frac{63}{16}$ **(e)** $\frac{24}{25}$ **(f)** $-\frac{119}{169}$ **(g)** $\frac{2\sqrt{13}}{13}$ **(h)** $-\frac{\sqrt{10}}{10}$ **87.** **(a)** $\frac{-\sqrt{3} - 2\sqrt{2}}{6}$ **(b)** $\frac{1 - 2\sqrt{6}}{6}$ **(c)** $\frac{-\sqrt{3} + 2\sqrt{2}}{6}$ **(d)** $\frac{8\sqrt{2} + 9\sqrt{3}}{23}$ **(e)** $-\frac{\sqrt{3}}{2}$ **(f)** $-\frac{7}{9}$ **(g)** $\frac{\sqrt{3}}{3}$ **(h)** $\frac{\sqrt{3}}{2}$ **89.** **(a)** 1 **(b)** 0 **(c)** $-\frac{1}{9}$ **(d)** Not defined **(e)** $\frac{4\sqrt{5}}{9}$ **(f)** $-\frac{1}{9}$ **(g)** $\frac{\sqrt{30}}{6}$ **(h)** $-\frac{\sqrt{6}\sqrt{3} - \sqrt{5}}{6}$ **91.** $\frac{4 + 3\sqrt{3}}{10}$