Concepts and Vocabulary

3. Suppose that f and g are two functions with the same domain. If $f(x) = g(x)$ for every x in the domain, the equation is called a(n) _____. Otherwise, it is called a(n) _____ equation.

4. $\tan^2\theta - \sec^2\theta =$ ____.

5. $\cos(-\theta) - \cos\theta =$ ____.

6. ***True or False*** $\sin(-\theta) + \sin\theta = 0$ for any value of θ.

7. ***True or False*** In establishing an identity, it is often easiest to just multiply both sides by a well-chosen nonzero expression involving the variable.

8. ***True or False*** $\tan\theta \cdot \cos\theta = \sin\theta$ for any $\theta \neq (2k+1)\frac{\pi}{2}$.

Skill Building

In Problems 9–18, simplify each trigonometric expression by following the indicated direction.

9. Rewrite in terms of sine and cosine functions: $\tan\theta \cdot \csc\theta$.

10. Rewrite in terms of sine and cosine functions: $\cot\theta \cdot \sec\theta$.

11. Multiply $\dfrac{\cos\theta}{1-\sin\theta}$ by $\dfrac{1+\sin\theta}{1+\sin\theta}$.

12. Multiply $\dfrac{\sin\theta}{1+\cos\theta}$ by $\dfrac{1-\cos\theta}{1-\cos\theta}$.

13. Rewrite over a common denominator:
$$\frac{\sin\theta+\cos\theta}{\cos\theta} + \frac{\cos\theta-\sin\theta}{\sin\theta}$$

14. Rewrite over a common denominator:
$$\frac{1}{1-\cos v} + \frac{1}{1+\cos v}$$

15. Multiply and simplify: $\dfrac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)-1}{\sin\theta\cos\theta}$

16. Multiply and simplify: $\dfrac{(\tan\theta+1)(\tan\theta+1)-\sec^2\theta}{\tan\theta}$

17. Factor and simplify: $\dfrac{3\sin^2\theta+4\sin\theta+1}{\sin^2\theta+2\sin\theta+1}$

18. Factor and simplify: $\dfrac{\cos^2\theta-1}{\cos^2\theta-\cos\theta}$

In Problems 19–98, establish each identity.

19. $\csc\theta \cdot \cos\theta = \cot\theta$

20. $\sec\theta \cdot \sin\theta = \tan\theta$

21. $1 + \tan^2(-\theta) = \sec^2\theta$

22. $1 + \cot^2(-\theta) = \csc^2\theta$

23. $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$

24. $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

25. $\tan u \cot u - \cos^2 u = \sin^2 u$

26. $\sin u \csc u - \cos^2 u = \sin^2 u$

27. $(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$

28. $(\csc\theta - 1)(\csc\theta + 1) = \cot^2\theta$

29. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

30. $(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

31. $\cos^2\theta(1 + \tan^2\theta) = 1$

32. $(1 - \cos^2\theta)(1 + \cot^2\theta) = 1$

33. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

34. $\tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta = 1$

35. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

36. $\csc^4\theta - \csc^2\theta = \cot^4\theta + \cot^2\theta$

37. $\sec u - \tan u = \dfrac{\cos u}{1+\sin u}$

38. $\csc u - \cot u = \dfrac{\sin u}{1+\cos u}$

39. $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$

40. $9\sec^2\theta - 5\tan^2\theta = 5 + 4\sec^2\theta$

41. $1 - \dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$

42. $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

43. $\dfrac{1+\tan v}{1-\tan v} = \dfrac{\cot v+1}{\cot v-1}$

44. $\dfrac{\csc v-1}{\csc v+1} = \dfrac{1-\sin v}{1+\sin v}$

45. $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = 2\tan\theta$

46. $\dfrac{\csc\theta-1}{\cot\theta} = \dfrac{\cot\theta}{\csc\theta+1}$

47. $\dfrac{1+\sin\theta}{1-\sin\theta} = \dfrac{\csc\theta+1}{\csc\theta-1}$

48. $\dfrac{\cos\theta+1}{\cos\theta-1} = \dfrac{1+\sec\theta}{1-\sec\theta}$

49. $\dfrac{1-\sin v}{\cos v} + \dfrac{\cos v}{1-\sin v} = 2\sec v$

50. $\dfrac{\cos v}{1+\sin v} + \dfrac{1+\sin v}{\cos v} = 2\sec v$

51. $\dfrac{\sin\theta}{\sin\theta-\cos\theta} = \dfrac{1}{1-\cot\theta}$

52. $1 - \dfrac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

53. $\dfrac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

54. $\dfrac{1-\cos\theta}{1+\cos\theta} = (\csc\theta - \cot\theta)^2$

55. $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$

56. $\dfrac{\cot\theta}{1-\tan\theta} + \dfrac{\tan\theta}{1-\cot\theta} = 1 + \tan\theta + \cot\theta$

57. $\tan\theta + \dfrac{\cos\theta}{1+\sin\theta} = \sec\theta$

58. $\dfrac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{\tan\theta}{1-\tan^2\theta}$

59. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

60. $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{\sin\theta + 1}{\cos\theta}$

61. $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = \sin^2\theta - \cos^2\theta$

62. $\dfrac{\sec\theta - \cos\theta}{\sec\theta + \cos\theta} = \dfrac{\sin^2\theta}{1+\cos^2\theta}$

63. $\dfrac{\tan u - \cot u}{\tan u + \cot u} + 1 = 2\sin^2 u$

64. $\dfrac{\tan u - \cot u}{\tan u + \cot u} + 2\cos^2 u = 1$

65. $\dfrac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta} = \tan\theta\sec\theta$

66. $\dfrac{\sec\theta}{1+\sec\theta} = \dfrac{1-\cos\theta}{\sin^2\theta}$

67. $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} + 1 = 2\cos^2\theta$

68. $\dfrac{1-\cot^2\theta}{1+\cot^2\theta} + 2\cos^2\theta = 1$

69. $\dfrac{\sec\theta - \csc\theta}{\sec\theta\csc\theta} = \sin\theta - \cos\theta$

70. $\dfrac{\sin^2\theta - \tan\theta}{\cos^2\theta - \cot\theta} = \tan^2\theta$

71. $\sec\theta - \cos\theta = \sin\theta\tan\theta$

72. $\tan\theta + \cot\theta = \sec\theta\csc\theta$

73. $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$

74. $\dfrac{1+\sin\theta}{1-\sin\theta} - \dfrac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$

75. $\dfrac{\sec\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos^3\theta}$

76. $\dfrac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$

77. $\dfrac{(\sec v - \tan v)^2 + 1}{\csc v(\sec v - \tan v)} = 2\tan v$

78. $\dfrac{\sec^2 v - \tan^2 v + \tan v}{\sec v} = \sin v + \cos v$

79. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} - \dfrac{\sin\theta - \cos\theta}{\sin\theta} = \sec\theta\csc\theta$

80. $\dfrac{\sin\theta + \cos\theta}{\sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta} = \sec\theta\csc\theta$

81. $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta\cos\theta$

82. $\dfrac{\sin^3\theta + \cos^3\theta}{1 - 2\cos^2\theta} = \dfrac{\sec\theta - \sin\theta}{\tan\theta - 1}$

83. $\dfrac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \cos^2\theta$

84. $\dfrac{\cos\theta + \sin\theta - \sin^3\theta}{\sin\theta} = \cot\theta + \cos^2\theta$

85. $\dfrac{(2\cos^2\theta - 1)^2}{\cos^4\theta - \sin^4\theta} = 1 - 2\sin^2\theta$

86. $\dfrac{1 - 2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$

87. $\dfrac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

88. $\dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta} = \sec\theta + \tan\theta$

89. $(a\sin\theta + b\cos\theta)^2 + (a\cos\theta - b\sin\theta)^2 = a^2 + b^2$

90. $(2a\sin\theta\cos\theta)^2 + a^2(\cos^2\theta - \sin^2\theta)^2 = a^2$

91. $\dfrac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \tan\alpha\tan\beta$

92. $(\tan\alpha + \tan\beta)(1 - \cot\alpha\cot\beta) + (\cot\alpha + \cot\beta)(1 - \tan\alpha\tan\beta) = 0$

93. $(\sin\alpha + \cos\beta)^2 + (\cos\beta + \sin\alpha)(\cos\beta - \sin\alpha) = 2\cos\beta(\sin\alpha + \cos\beta)$

94. $(\sin\alpha - \cos\beta)^2 + (\cos\beta + \sin\alpha)(\cos\beta - \sin\alpha) = -2\cos\beta(\sin\alpha - \cos\beta)$

95. $\ln|\sec\theta| = -\ln|\cos\theta|$

96. $\ln|\tan\theta| = \ln|\sin\theta| - \ln|\cos\theta|$

97. $\ln|1 + \cos\theta| + \ln|1 - \cos\theta| = 2\ln|\sin\theta|$

98. $\ln|\sec\theta + \tan\theta| + \ln|\sec\theta - \tan\theta| = 0$

In Problems 99–102, show that the functions f and g are identically equal.

99. $f(x) = \sin x \cdot \tan x \quad g(x) = \sec x - \cos x$

100. $f(x) = \cos x \cdot \cot x \quad g(x) = \csc x - \sin x$

101. $f(\theta) = \dfrac{1-\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} \quad g(\theta) = 0$

102. $f(\theta) = \tan\theta + \sec\theta \quad g(\theta) = \dfrac{\cos\theta}{1-\sin\theta}$