

The matrix is now in echelon form. The final matrix represents the system

$$\begin{cases} f + y + r = 46 & (1) \\ y - 2r = 2 & (2) \\ r = 1 & (3) \end{cases}$$

From equation (3), we determine that 1 red card was given. Back-substitute  $r = 1$  into equation (2) to find that  $y = 4$ , so 4 yellow cards were given. Back-substitute these values into equation (1) and to find that  $f = 41$ , so 41 fouls were called.



**COMMENT** Most graphing utilities have the capability to put an augmented matrix into echelon form (ref) and also reduced row echelon form (rref). See the Appendix, Section 7, for a discussion.

## 12.2 Assess Your Understanding

### Concepts and Vocabulary

- An  $m$  by  $n$  rectangular array of numbers is called a(n) \_\_\_\_\_.
- The matrix used to represent a system of linear equations is called a(n) \_\_\_\_\_ matrix.
- True or False** The augmented matrix of a system of two equations containing three variables has two rows and four columns.
- True or False** The matrix  $\left[ \begin{array}{cc|c} 1 & 3 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{array} \right]$  is in row echelon form.

### Skill Building

In Problems 5–16, write the augmented matrix of the given system of equations.

5.  $\begin{cases} x - 5y = 5 \\ 4x + 3y = 6 \end{cases}$

6.  $\begin{cases} 3x + 4y = 7 \\ 4x - 2y = 5 \end{cases}$

7.  $\begin{cases} 2x + 3y - 6 = 0 \\ 4x - 6y + 2 = 0 \end{cases}$

8.  $\begin{cases} 9x - y = 0 \\ 3x - y - 4 = 0 \end{cases}$

9.  $\begin{cases} 0.01x - 0.03y = 0.06 \\ 0.13x + 0.10y = 0.20 \end{cases}$

10.  $\begin{cases} \frac{4}{3}x - \frac{3}{2}y = \frac{3}{4} \\ -\frac{1}{4}x + \frac{1}{3}y = \frac{2}{3} \end{cases}$

11.  $\begin{cases} x - y + z = 10 \\ 3x + 3y = 5 \\ x + y + 2z = 2 \end{cases}$

12.  $\begin{cases} 5x - y - z = 0 \\ x + y = 5 \\ 2x - 3z = 2 \end{cases}$

13.  $\begin{cases} x + y - z = 2 \\ 3x - 2y = 2 \\ 5x + 3y - z = 1 \end{cases}$

14.  $\begin{cases} 2x + 3y - 4z = 0 \\ x - 5z + 2 = 0 \\ x + 2y - 3z = -2 \end{cases}$

15.  $\begin{cases} x - y - z = 10 \\ 2x + y + 2z = -1 \\ -3x + 4y = 5 \\ 4x - 5y + z = 0 \end{cases}$

16.  $\begin{cases} x - y + 2z - w = 5 \\ x + 3y - 4z + 2w = 2 \\ 3x - y - 5z - w = -1 \end{cases}$

In Problems 17–24, write the system of equations corresponding to each augmented matrix. Then perform each row operation on the given augmented matrix.

17.  $\left[ \begin{array}{cc|c} 1 & -3 & -2 \\ 2 & -5 & 5 \end{array} \right] R_2 = -2r_1 + r_2$

18.  $\left[ \begin{array}{cc|c} 1 & -3 & -3 \\ 2 & -5 & -4 \end{array} \right] R_2 = -2r_1 + r_2$

19.  $\left[ \begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 3 & -5 & 6 & 6 \\ -5 & 3 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_2 = -3r_1 + r_2 \\ \text{(b) } R_3 = 5r_1 + r_3 \end{array}$

20.  $\left[ \begin{array}{ccc|c} 1 & -3 & 3 & -5 \\ -4 & -5 & -3 & -5 \\ -3 & -2 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_2 = 4r_1 + r_2 \\ \text{(b) } R_3 = 3r_1 + r_3 \end{array}$

21.  $\left[ \begin{array}{ccc|c} 1 & -3 & 2 & -6 \\ 2 & -5 & 3 & -4 \\ -3 & -6 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_2 = -2r_1 + r_2 \\ \text{(b) } R_3 = 3r_1 + r_3 \end{array}$

22.  $\left[ \begin{array}{ccc|c} 1 & -3 & -4 & -6 \\ 6 & -5 & 6 & -6 \\ -1 & 1 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_2 = -6r_1 + r_2 \\ \text{(b) } R_3 = r_1 + r_3 \end{array}$

23.  $\left[ \begin{array}{ccc|c} 5 & -3 & 1 & -2 \\ 2 & -5 & 6 & -2 \\ -4 & 1 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_1 = -2r_2 + r_1 \\ \text{(b) } R_1 = r_3 + r_1 \end{array}$

24.  $\left[ \begin{array}{ccc|c} 4 & -3 & -1 & 2 \\ 3 & -5 & 2 & 6 \\ -3 & -6 & 4 & 6 \end{array} \right] \begin{array}{l} \text{(a) } R_1 = -r_2 + r_1 \\ \text{(b) } R_1 = r_3 + r_1 \end{array}$

In Problems 25–36, the reduced row echelon form of a system of linear equations is given. Write the system of equations corresponding to the given matrix. Use  $x$ ,  $y$ ; or  $x$ ,  $y$ ,  $z$ ; or  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$  as variables. Determine whether the system is consistent or inconsistent. If it is consistent, give the solution.

$$25. \left[ \begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -1 \end{array} \right]$$

$$26. \left[ \begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 0 \end{array} \right]$$

$$27. \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 \end{array} \right]$$

$$28. \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

$$29. \left[ \begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$30. \left[ \begin{array}{ccc|c} 1 & 0 & 4 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$31. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right]$$

$$32. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 3 & 0 \end{array} \right]$$

$$33. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 4 & 2 \\ 0 & 1 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$34. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 3 \end{array} \right]$$

$$35. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$36. \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

In Problems 37–72, solve each system of equations using matrices (row operations). If the system has no solution, say that it is inconsistent.

$$37. \begin{cases} x + y = 8 \\ x - y = 4 \end{cases}$$

$$38. \begin{cases} x + 2y = 5 \\ x + y = 3 \end{cases}$$

$$39. \begin{cases} 2x - 4y = -2 \\ 3x + 2y = 3 \end{cases}$$

$$40. \begin{cases} 3x + 3y = 3 \\ 4x + 2y = \frac{8}{3} \end{cases}$$

$$41. \begin{cases} x + 2y = 4 \\ 2x + 4y = 8 \end{cases}$$

$$42. \begin{cases} 3x - y = 7 \\ 9x - 3y = 21 \end{cases}$$

$$43. \begin{cases} 2x + 3y = 6 \\ x - y = \frac{1}{2} \end{cases}$$

$$44. \begin{cases} \frac{1}{2}x + y = -2 \\ x - 2y = 8 \end{cases}$$

$$45. \begin{cases} 3x - 5y = 3 \\ 15x + 5y = 21 \end{cases}$$

$$46. \begin{cases} 2x - y = -1 \\ x + \frac{1}{2}y = \frac{3}{2} \end{cases}$$

$$47. \begin{cases} x - y = 6 \\ 2x - 3z = 16 \\ 2y + z = 4 \end{cases}$$

$$48. \begin{cases} 2x + y = -4 \\ -2y + 4z = 0 \\ 3x - 2z = -11 \end{cases}$$

$$49. \begin{cases} x - 2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

$$50. \begin{cases} 2x + y - 3z = 0 \\ -2x + 2y + z = -7 \\ 3x - 4y - 3z = 7 \end{cases}$$

$$51. \begin{cases} 2x - 2y - 2z = 2 \\ 2x + 3y + z = 2 \\ 3x + 2y = 0 \end{cases}$$

$$52. \begin{cases} 2x - 3y - z = 0 \\ -x + 2y + z = 5 \\ 3x - 4y - z = 1 \end{cases}$$

$$53. \begin{cases} -x + y + z = -1 \\ -x + 2y - 3z = -4 \\ 3x - 2y - 7z = 0 \end{cases}$$

$$54. \begin{cases} 2x - 3y - z = 0 \\ 3x + 2y + 2z = 2 \\ x + 5y + 3z = 2 \end{cases}$$

$$55. \begin{cases} 2x - 2y + 3z = 6 \\ 4x - 3y + 2z = 0 \\ -2x + 3y - 7z = 1 \end{cases}$$

$$56. \begin{cases} 3x - 2y + 2z = 6 \\ 7x - 3y + 2z = -1 \\ 2x - 3y + 4z = 0 \end{cases}$$

$$57. \begin{cases} x + y - z = 6 \\ 3x - 2y + z = -5 \\ x + 3y - 2z = 14 \end{cases}$$

$$58. \begin{cases} x - y + z = -4 \\ 2x - 3y + 4z = -15 \\ 5x + y - 2z = 12 \end{cases}$$

$$59. \begin{cases} x + 2y - z = -3 \\ 2x - 4y + z = -7 \\ -2x + 2y - 3z = 4 \end{cases}$$

$$60. \begin{cases} x + 4y - 3z = -8 \\ 3x - y + 3z = 12 \\ x + y + 6z = 1 \end{cases}$$

$$61. \begin{cases} 3x + y - z = \frac{2}{3} \\ 2x - y + z = 1 \\ 4x + 2y = \frac{8}{3} \end{cases}$$

$$62. \begin{cases} x + y = 1 \\ 2x - y + z = 1 \\ x + 2y + z = \frac{8}{3} \end{cases}$$

$$63. \begin{cases} x + y + z + w = 4 \\ 2x - y + z = 0 \\ 3x + 2y + z - w = 6 \\ x - 2y - 2z + 2w = -1 \end{cases}$$

$$64. \begin{cases} x + y + z + w = 4 \\ -x + 2y + z = 0 \\ 2x + 3y + z - w = 6 \\ -2x + y - 2z + 2w = -1 \end{cases}$$

$$65. \begin{cases} x + 2y + z = 1 \\ 2x - y + 2z = 2 \\ 3x + y + 3z = 3 \end{cases}$$

$$66. \begin{cases} x + 2y - z = 3 \\ 2x - y + 2z = 6 \\ x - 3y + 3z = 4 \end{cases}$$

$$67. \begin{cases} x - y + z = 5 \\ 3x + 2y - 2z = 0 \end{cases}$$

$$68. \begin{cases} 2x + y - z = 4 \\ -x + y + 3z = 1 \end{cases}$$

$$69. \begin{cases} 2x + 3y - z = 3 \\ x - y - z = 0 \\ -x + y + z = 0 \\ x + y + 3z = 5 \end{cases}$$

$$70. \begin{cases} x - 3y + z = 1 \\ 2x - y - 4z = 0 \\ x - 3y + 2z = 1 \\ x - 2y = 5 \end{cases}$$

$$71. \begin{cases} 4x + y + z - w = 4 \\ x - y + 2z + 3w = 3 \end{cases}$$

$$72. \begin{cases} -4x + y = 5 \\ 2x - y + z - w = 5 \\ z + w = 4 \end{cases}$$

### Applications and Extensions

- 73. Curve Fitting** Find the function  $y = ax^2 + bx + c$  whose graph contains the points  $(1, 2)$ ,  $(-2, -7)$ , and  $(2, -3)$ .
- 74. Curve Fitting** Find the function  $y = ax^2 + bx + c$  whose graph contains the points  $(1, -1)$ ,  $(3, -1)$ , and  $(-2, 14)$ .
- 75. Curve Fitting** Find the function  $f(x) = ax^3 + bx^2 + cx + d$  for which  $f(-3) = -112$ ,  $f(-1) = -2$ ,  $f(1) = 4$ , and  $f(2) = 13$ .
- 76. Curve Fitting** Find the function  $f(x) = ax^3 + bx^2 + cx + d$  for which  $f(-2) = -10$ ,  $f(-1) = 3$ ,  $f(1) = 5$ , and  $f(3) = 15$ .
- 77. Nutrition** A dietitian at Palos Community Hospital wants a patient to have a meal that has 78 grams of protein, 59 grams of carbohydrates, and 75 milligrams of vitamin A. The hospital food service tells the dietitian that the dinner for today is salmon steak, baked eggs, and acorn squash. Each serving of salmon steak has 30 grams of protein, 20 grams of carbohydrates, and 2 milligrams of vitamin A. Each serving of baked eggs contains 15 grams of protein, 2 grams of carbohydrates, and 20 milligrams of vitamin A. Each serving of acorn squash contains 3 grams of protein, 25 grams of carbohydrates, and 32 milligrams of vitamin A. How many servings of each food should the dietitian provide for the patient?
- 78. Nutrition** A dietitian at General Hospital wants a patient to have a meal that has 47 grams of protein, 58 grams of carbohydrates, and 630 milligrams of calcium. The hospital food service tells the dietitian that the dinner for today is pork chops, corn on the cob, and 2% milk. Each serving of pork chops has 23 grams of protein, 0 grams of carbohydrates, and 10 milligrams of calcium. Each serving of corn on the cob contains 3 grams of protein, 16 grams of carbohydrates, and 10 milligrams of calcium. Each glass of 2% milk contains 9 grams of protein, 13 grams of carbohydrates, and 300 milligrams of calcium. How many servings of each food should the dietitian provide for the patient?
- 79. Financial Planning** Carletta has \$10,000 to invest. As her financial consultant, you recommend that she invest in Treasury bills that yield 6%, Treasury bonds that yield 7%, and corporate bonds that yield 8%. Carletta wants to have an annual income of \$680, and the amount invested in corporate bonds must be half that invested in Treasury bills. Find the amount in each investment.
- 80. Landscaping** A landscape company is hired to plant trees in three new subdivisions. The company charges the developer for each tree planted, an hourly rate to plant the trees, and a fixed delivery charge. In one subdivision it took 166 labor hours to plant 250 trees for a cost of \$7520. In a second subdivision it took 124 labor hours to plant 200 trees for a cost of \$5945. In the final subdivision it took 200 labor hours to plant 300 trees for a cost of \$8985. Determine the cost for each tree, the hourly labor charge, and the fixed delivery charge.  
*Sources: gurney.com; www.bx.org*
- 81. Production** To manufacture an automobile requires painting, drying, and polishing. Epsilon Motor Company produces three types of cars: the Delta, the Beta, and the Sigma. Each Delta requires 10 hours for painting, 3 hours for drying, and 2 hours for polishing. A Beta requires 16 hours for painting, 5 hours for drying, and 3 hours for polishing, and a Sigma requires 8 hours for painting, 2 hours for drying, and 1 hour for polishing. If the company has 240 hours for painting, 69 hours for drying, and 41 hours for polishing per month, how many of each type of car are produced?
- 82. Production** A Florida juice company completes the preparation of its products by sterilizing, filling, and labeling bottles. Each case of orange juice requires 9 minutes for sterilizing, 6 minutes for filling, and 1 minute for labeling. Each case of grapefruit juice requires 10 minutes for sterilizing, 4 minutes for filling, and 2 minutes for labeling. Each case of tomato juice requires 12 minutes for sterilizing, 4 minutes for filling, and 1 minute for labeling. If the company runs the sterilizing machine for 398 minutes, the filling machine for 164 minutes, and the labeling machine for 58 minutes, how many cases of each type of juice are prepared?
- 83. Electricity: Kirchhoff's Rules** An application of Kirchhoff's Rules to the circuit shown results in the following system of equations:
- $$\begin{cases} -4 + 8 - 2I_2 = 0 \\ 8 = 5I_4 + I_1 \\ 4 = 3I_3 + I_1 \\ I_3 + I_4 = I_1 \end{cases}$$

Find the currents  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$ .

47.  $\{(x, y, z) \mid x = 5z - 2, y = 4z - 3; z \text{ is any real number}\}$  49. Inconsistent 51.  $x = 1, y = 3, z = -2; (1, 3, -2)$   
 53.  $x = -3, y = \frac{1}{2}, z = 1; (-3, \frac{1}{2}, 1)$  55. Length 30 ft; width 15 ft 57. There were 18 commercial launches and 37 noncommercial launches in 2005.  
 59. 22.5 lb 61. Average wind speed 25 mph; average airspeed 175 mph 63. 80 \$25 sets and 120 \$45 sets 65. \$9.96 67. Mix 50 mg of first compound with 75 mg of second. 69.  $a = \frac{4}{3}, b = -\frac{5}{3}, c = 1$  71.  $y = 9000, r = 0.06$  73.  $I_1 = \frac{10}{71}, I_2 = \frac{65}{71}, I_3 = \frac{55}{71}$  75. 100 orchestra, 210 main, and 190 balcony seats 77. 1.5 chicken, 1 corn, 2 milk

79. If  $x$  = price of hamburgers,  $y$  = price of fries, and  $z$  = price of colas, then

<b>x</b>	\$2.13	\$2.01	\$1.86
<b>y</b>	\$0.89	\$0.93	\$0.98
<b>z</b>	\$0.62	\$0.74	\$0.89

$x = 2.75 - z, y = \frac{41}{60} + \frac{1}{3}z, \$0.60 \leq z \leq \$0.90.$

There is not sufficient information:

81. It will take Beth 30 hr, Bill 24 hr, and Edie 40 hr.

### 12.2 Assess Your Understanding (page 862)

1. matrix 2. augmented 3. T 4. T

5.  $\begin{bmatrix} 1 & -5 & 5 \\ 4 & 3 & 6 \end{bmatrix}$  7.  $\begin{bmatrix} 2 & 3 & 6 \\ 4 & -6 & -2 \end{bmatrix}$  9.  $\begin{bmatrix} 0.01 & -0.03 & 0.06 \\ 0.13 & 0.10 & 0.20 \end{bmatrix}$  11.  $\begin{bmatrix} 1 & -1 & 1 & 10 \\ 3 & 3 & 0 & 5 \\ 1 & 1 & 2 & 2 \end{bmatrix}$  13.  $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 3 & -2 & 0 & 2 \\ 5 & 3 & -1 & 1 \end{bmatrix}$  15.  $\begin{bmatrix} 1 & -1 & -1 & 10 \\ 2 & 1 & 2 & -1 \\ -3 & 4 & 0 & 5 \\ 4 & -5 & 1 & 0 \end{bmatrix}$

17.  $\begin{cases} x - 3y = -2 & (1) \\ 2x - 5y = 5 & (2) \end{cases}$   $\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 9 \end{bmatrix}$  19.  $\begin{cases} x - 3y + 4z = 3 & (1) \\ 3x - 5y + 6z = 6 & (2) \\ -5x + 3y + 4z = 6 & (3) \end{cases}$  (a)  $\begin{bmatrix} 1 & -3 & 4 & 3 \\ 0 & 4 & -6 & -3 \\ -5 & 3 & 4 & 6 \end{bmatrix}$  (b)  $\begin{bmatrix} 1 & -3 & 4 & 3 \\ 3 & -5 & 6 & 6 \\ 0 & -12 & 24 & 21 \end{bmatrix}$

21.  $\begin{cases} x - 3y + 2z = -6 & (1) \\ 2x - 5y + 3z = -4 & (2) \\ -3x - 6y + 4z = 6 & (3) \end{cases}$  (a)  $\begin{bmatrix} 1 & -3 & 2 & -6 \\ 0 & 1 & -1 & 8 \\ -3 & -6 & 4 & 6 \end{bmatrix}$  (b)  $\begin{bmatrix} 1 & -3 & 2 & -6 \\ 2 & -5 & 3 & -4 \\ 0 & -15 & 10 & -12 \end{bmatrix}$

23.  $\begin{cases} 5x - 3y + z = -2 & (1) \\ 2x - 5y + 6z = -2 & (2) \\ -4x + y + 4z = 6 & (3) \end{cases}$  (a)  $\begin{bmatrix} 1 & 7 & -11 & 2 \\ 2 & -5 & 6 & -2 \\ -4 & 1 & 4 & 6 \end{bmatrix}$  (b)  $\begin{bmatrix} 1 & -2 & 5 & 4 \\ 2 & -5 & 6 & -2 \\ -4 & 1 & 4 & 6 \end{bmatrix}$  25.  $\begin{cases} x = 5 \\ y = -1 \end{cases}$  27.  $\begin{cases} x = 1 \\ y = 2 \\ z = 0 \end{cases}$

Consistent:  $x = 5, y = -1$  or  $(5, -1)$  Inconsistent

29.  $\begin{cases} x + 2z = -1 \\ y - 4z = -2 \\ 0 = 0 \end{cases}$

Consistent:  
 $\begin{cases} x = -1 - 2z \\ y = -2 + 4z \\ z \text{ is any real number or} \\ \{(x, y, z) \mid x = -1 - 2z, \\ y = -2 + 4z, z \text{ is any real} \\ \text{number}\} \end{cases}$

31.  $\begin{cases} x_1 = 1 \\ x_2 + x_4 = 2 \\ x_3 + 2x_4 = 3 \end{cases}$

Consistent:  
 $\begin{cases} x_1 = 1, x_2 = 2 - x_4 \\ x_3 = 3 - 2x_4 \\ x_4 \text{ is any real number or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = 1, \\ x_2 = 2 - x_4, x_3 = 3 - 2x_4, \\ x_4 \text{ is any real number}\} \end{cases}$

33.  $\begin{cases} x_1 + 4x_4 = 2 \\ x_2 + x_3 + 3x_4 = 3 \\ 0 = 0 \end{cases}$

Consistent:  
 $\begin{cases} x_1 = 2 - 4x_4 \\ x_2 = 3 - x_3 - 3x_4 \\ x_3, x_4 \text{ are any real numbers or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = 2 - 4x_4, \\ x_2 = 3 - x_4 - 3x_4, x_3, x_4 \text{ are} \\ \text{any real numbers}\} \end{cases}$

35.  $\begin{cases} x_1 + x_4 = -2 \\ x_2 + 2x_4 = 2 \\ x_3 - x_4 = 0 \end{cases}$

Consistent:  
 $\begin{cases} x_1 = -2 - x_4 \\ x_2 = 2 - 2x_4 \\ x_3 = x_4 \\ x_4 \text{ is any real number or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = -2 - x_4, \\ x_2 = 2 - 2x_4, x_3 = x_4, x_4 \text{ is any} \\ \text{real number}\} \end{cases}$

37.  $x = 6, y = 2; (6, 2)$  39.  $x = \frac{1}{2}, y = \frac{3}{4}; (\frac{1}{2}, \frac{3}{4})$  41.  $x = 4 - 2y, y \text{ is any real number}; \{(x, y) \mid x = 4 - 2y, y \text{ is any real number}\}$

43.  $x = \frac{3}{2}, y = 1; (\frac{3}{2}, 1)$  45.  $x = \frac{4}{3}, y = \frac{1}{5}; (\frac{4}{3}, \frac{1}{5})$  47.  $x = 8, y = 2, z = 0; (8, 2, 0)$  49.  $x = 2, y = -1, z = 1; (2, -1, 1)$  51. Inconsistent

53.  $x = 5z - 2, y = 4z - 3, z \text{ is any real number}; \{(x, y, z) \mid x = 5z - 2, y = 4z - 3, z \text{ is any real number}\}$  55. Inconsistent

57.  $x = 1, y = 3, z = -2; (1, 3, -2)$  59.  $x = -3, y = \frac{1}{2}, z = 1; (-3, \frac{1}{2}, 1)$  61.  $x = \frac{1}{3}, y = \frac{2}{3}, z = 1; (\frac{1}{3}, \frac{2}{3}, 1)$  63.  $x = 1, y = 2, z = 0, w = 1;$

$(1, 2, 0, 1)$  65.  $y = 0, z = 1 - x, x \text{ is any real number or } \{(x, y, z) \mid y = 0, z = 1 - x, x \text{ is any real number}\}$  67.  $x = 2, y = z - 3, z \text{ is any}$

real number or  $\{(x, y, z) \mid x = 2, y = z - 3, z \text{ is any real number}\}$  69.  $x = \frac{13}{9}, y = \frac{7}{18}, z = \frac{19}{18}; (\frac{13}{9}, \frac{7}{18}, \frac{19}{18})$

71.  $x = \frac{7}{5} - \frac{3}{5}z - \frac{2}{5}w, y = -\frac{8}{5} + \frac{7}{5}z + \frac{13}{5}w, z \text{ and } w \text{ are any real numbers or } \{(x, y, z, w) \mid x = \frac{7}{5} - \frac{3}{5}z - \frac{2}{5}w, y = -\frac{8}{5} + \frac{7}{5}z + \frac{13}{5}w,$

$z \text{ and } w \text{ are any real numbers}\}$  73.  $y = -2x^2 + x + 3$  75.  $f(x) = 3x^3 - 4x^2 + 5$  77. 1.5 salmon steak, 2 baked eggs, 1 acorn squash

79. \$4000 in Treasury bills, \$4000 in Treasury bonds, \$2000 in corporate bonds 81. 8 Deltas, 5 Betas, 10 Sigmas 83.  $I_1 = \frac{44}{23}, I_2 = 2, I_3 = \frac{16}{23}, I_4 = \frac{28}{23}$