

# Math Honors 2: Portfolio Task 1 (due at 8:30 a.m. on Dec. 15)

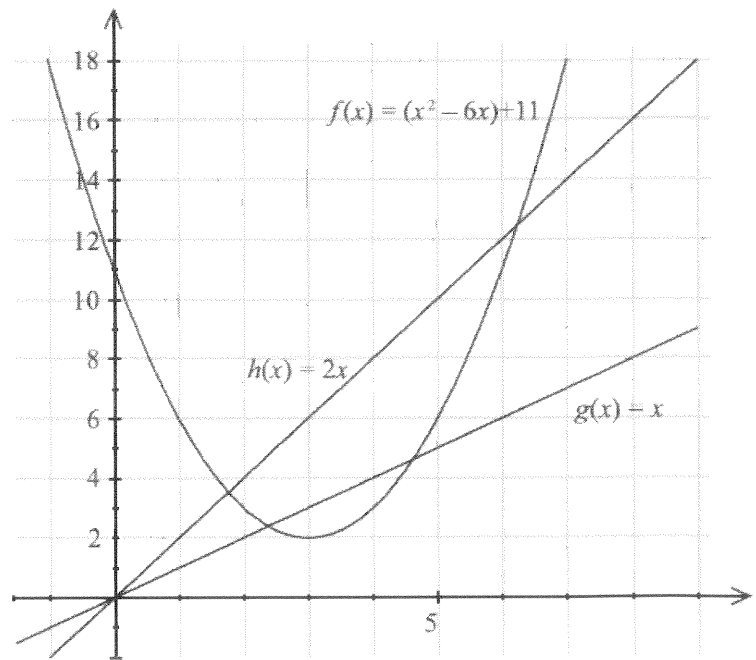
## PARABOLA INVESTIGATION

### Description

In this task, you will investigate the patterns in the intersections of parabolas and the lines  $y = x$  and  $y = 2x$ . Then you will be asked to prove your conjectures and to broaden the scope of the investigation to include other lines and other types of polynomials.

### Method

1. Consider the parabola  $y = (x-3)^2 + 2 = x^2 - 6x + 11$  and the lines  $y = x$  and  $y = 2x$ .
  - Using technology find the four intersections illustrated on the right.
  - Label the  $x$ -values of these intersections as they appear from left to right on the  $x$ -axis as  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$ .
  - Find the values of  $x_2 - x_1$  and  $x_4 - x_3$  and name them respectively  $S_L$  and  $S_R$ .
  - Finally, calculate  $D = |S_L - S_R|$ .



2. Find values of  $D$  for other parabolas of the form  $y = ax^2 + bx + c$ ,  $a > 0$ , with vertices in quadrant 1, intersected by the lines  $y = x$  and  $y = 2x$ . Consider various values of  $a$ , beginning with  $a = 1$ . Make a conjecture about the value of  $D$  for these parabolas.
3. Investigate your conjecture for any real value of  $a$  and any placement of the vertex. Refine your conjecture as necessary, and prove it. Maintain the labeling convention used in parts 1 and 2 by having the intersections of the first line to be  $x_1$  and  $x_3$  and the intersections with the second line to be  $x_2$  and  $x_4$ .
4. Does your conjecture hold if the intersecting lines are changed? Modify your conjecture, if necessary, and prove it.
5. Determine whether a similar conjecture can be made for cubic polynomials.
6. Consider whether the conjecture might be modified to include higher order polynomials.

Overview of assessment criteria for type I tasks

	Criterion A: Use of notation and terminology	Criterion B: Communication	Criterion C: Mathematical process — searching for patterns	Criterion D: Results — generalization	Criterion E: Use of technology	Criterion F: Quality of work
0	The student does not use appropriate notation and terminology.	The student neither provides explanations nor uses appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student does not attempt to use a mathematical strategy.	The student does not produce any general statement consistent with the patterns and/or structures generated.	The student uses a calculator or computer for only routine calculations.	The student has shown a poor quality of work.
1	The student uses some appropriate notation and/or terminology.	The student attempts to provide explanations or uses some appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student uses a mathematical strategy to produce data.	The student attempts to produce a general statement that is consistent with the patterns and/or structures generated.	The student attempts to use a calculator or computer in a manner that could enhance the development of the task.	The student has shown a satisfactory quality of work.
2	The student uses appropriate notation and terminology in a consistent manner and does so throughout the work.	The student provides adequate explanations or arguments, and communicates them using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student organizes the data generated.	The student correctly produces a general statement that is consistent with the patterns and/or structures generated.	The student makes limited use of a calculator or computer in a manner that enhances the development of the task.	The student has shown an outstanding quality of work.
3		The student provides complete, coherent explanations or arguments, and communicates them clearly using appropriate forms of representation (for example, symbols, tables, graphs and/or diagrams).	The student attempts to analyse data to enable the formulation of a general statement.	The student expresses the correct general statement in appropriate mathematical terminology.	The student makes full and resourceful use of a calculator or computer in a manner that significantly enhances the development of the task.	
4			The student successfully analyses the correct data to enable the formulation of a general statement.	The student correctly states the scope or limitations of the general statement.		
5			The student tests the validity of the general statement by considering further examples.	The student gives a correct, informal justification of the general statement.		