

Name: _____ Original Score: _____ / 62 marks → _____ % → ISM: _____ → IB: _____
Date: _____ Block: _____ After Corrections: _____ / 62 marks

MATH HONORS 2: UNIT 4 TEST – Power, Radical, Exponential, and Logarithmic Functions

SECTION A: Calculator Inactive – 48 minutes

1. Explain why any non-zero number raised to the power of zero is equal to one. **(2 marks)**

2. Explain why the base of any exponential or logarithmic function must be positive, but not equal to one. **(2 marks)**

3. True or false: $(\ln 3)^2 = \ln 9 = 2\ln 3$. Explain your reasoning. **(2 marks)**

4. Given that $X = \log_a p$, $Y = \log_a q$, and $Z = \log_a r$, express $\log_a \left(\frac{p^2 r}{q^3} \right)$ in terms of X , Y , and Z . **(3 marks)**

5. Find the inverses of the following functions, stating any restrictions on the inverses' domains. **(6 marks)**

a. $f(x) = \sqrt{x+3} - 2$ **(3M)**

b. $g(x) = 3 \log_2(2-x) + 1$ **(3M)**

6. Simplify, expressing your final answers with only positive exponents. **(7 marks)**

a. $\frac{x^2(2x)^3 y^0}{4(x^{-4}y)^{-2}}$ **(4M)**

b. $\sqrt{50a^4b^2c^{-3}}$ **(3M)**

7. Solve for all real values of x : **(8 marks)**

a. $\sqrt{x} - \sqrt{x-3} > 1$ **(4M)**

b. $4^x - 2^{x+2} - 32 = 0$ **(4M)**

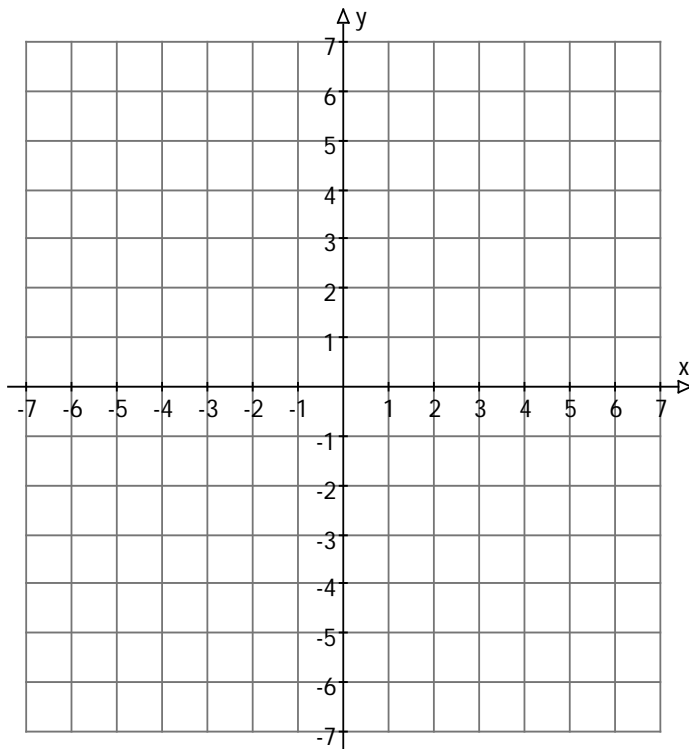
Name: _____

8. WonFine School opens in 1980, and its student population is modeled by the equation **(4 marks)**

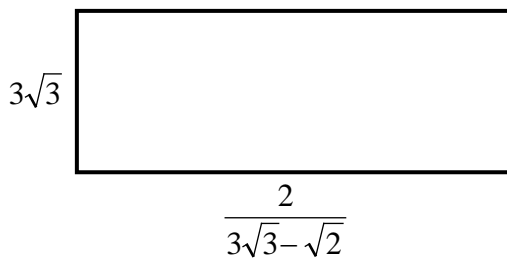
$$P(t) = 1620(0.995)^t + 250 \text{ where } t \text{ is the time in years.}$$

- a. Is the student population growing or declining? **(1M)**
- b. What is the annual rate of change (express as a percentage)? **(1M)**
- c. What is the initial population? **(1M)**
- d. What does the horizontal asymptote represent in the equation? **(1M)**

9. Sketch the function $f(x) = \ln(-x + 3)$ on the provided axes, clearly labeling any intercepts with coordinates and any asymptotes with equations. **(4 marks)**

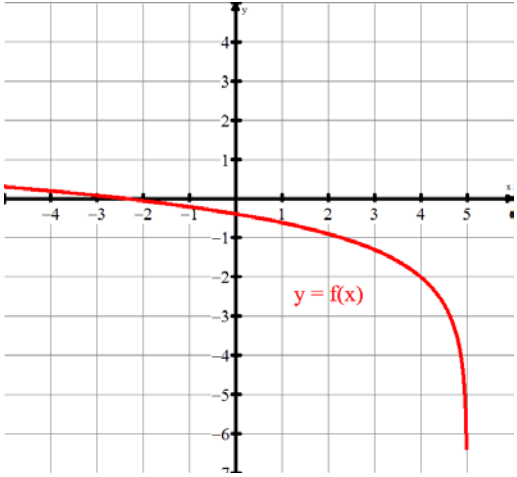
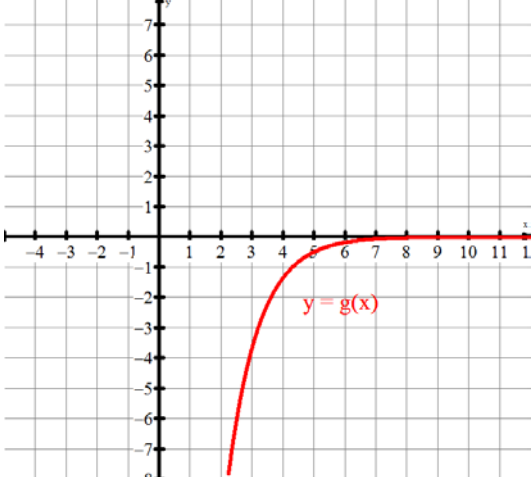
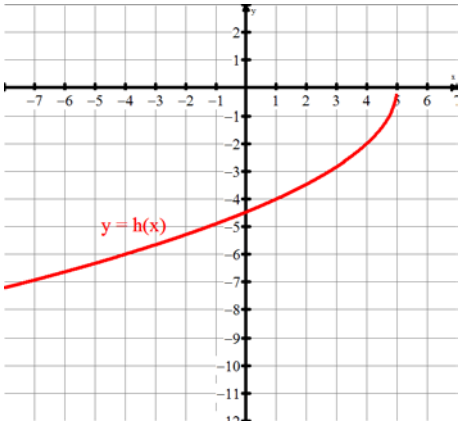
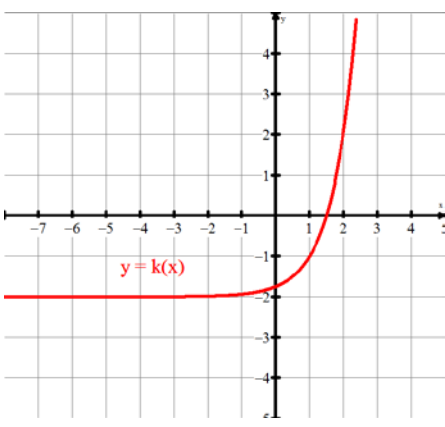


10. Determine the area of the given rectangle, simplifying your final answer. **(3 marks)**



11. Given the following graphs and equations, match the appropriate equation to the corresponding graph. Make sure your selection is obvious (i.e. write the chosen equation below each graph).

(4 marks)

$y(x) = -\frac{1}{2}e^{5-x}$	$y(x) = -2\sqrt{5-x}$	$y(x) = -\frac{1}{2}e^{5+x}$	$y(x) = -2 + \ln(5+x)$
$y(x) = -2 + 4^{1-x}$	$y(x) = -2 + \left(\frac{1}{4}\right)^{1-x}$	$y(x) = -2 + \ln(5-x)$	$y(x) = -2\sqrt{-5-x}$
<p style="text-align: center;">GRAPH A</p> 	<p style="text-align: center;">GRAPH B</p> 		
<p style="text-align: center;">GRAPH C</p> 	<p style="text-align: center;">GRAPH D</p> 		

SECTION B: Calculator Active – 17 minutes

12. Evaluate $\frac{\log_2 7}{\log_2 19}$ **(2 marks)**

13. Dry cleaners use a cleaning fluid that is purified by evaporation and condensation after each cleaning cycle. Every time it is purified, 3% of the fluid is lost. **(6 marks)**

a. An equipment manufacturer claims that after 15 cycles, at least two thirds of the cleaning fluid remains. Verify or reject this claim. **(2M)**

b. If the cleaning fluid has to be “topped up” when half of the original amount remains, after how many cleaning cycles should the fluid be “topped up”? **(2M)**

c. Another manufacturer has developed a new process such that two thirds of the fluid remains after 40 cleaning cycles. What percentage of cleaning fluid is lost each cycle? **(2M)**

14. The magnitude M of an earthquake is a unit-less number (also known as the Richter Scale) **(4 marks)**
that can be modeled by the equation $M = \frac{2}{3} \log M_0 - 10.7$, where M is always rounded to one decimal place. The seismic moment M_0 represents the total amount of energy that is transformed during an earthquake, and is measured in a unit called dyne centimeters.

a. What is the magnitude of an earthquake in South Africa that has a seismic moment of 8.42×10^{23} dyne centimeters? **(2M)**

b. What is the seismic moment if an earthquake in Chile measured 9.4 on the Richter Scale? Express your answer in scientific notation. **(2M)**

15. When you graduate, your parents present you with a large sum of money and allow you to choose between two investments. You can either deposit the money into account A which earns 30% per annum compounded semi-annually, or into account B which earns 29% per annum compounded continuously. Which investment would be more profitable? Explain your answer. **(3 marks)**

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STUDENT SELF-EVALUATION

After the time allocated for writing this assessment has passed (or if you have finished early), answer the following questions:

- a. Estimate the letter grade that you achieved on this assessment (e.g. A-, C+, etc.): _____

- b. Which concepts did you have the most difficulty with during this assessment and/or this unit?

COMMUNICATION

In every formal assessment this year, 2 marks, 1 mark, or 0 marks will be awarded for the clarity of your communication in the presentation of your solutions and your written explanations.

On this assessment, you were awarded: _____ / 2 marks for communication.