

Name: _____ Original Score: _____ / 62 marks → _____ % → ISM: _____ → IB: _____
Date: _____ Block: _____ After Corrections: _____ / 62 marks

MATH HONORS 2: UNIT 6 TEST – Matrices and Probability

SECTION A: Calculator Inactive – 40 minutes

1. A and B are 2 events such that $p(A) = 0.3$ and $p(B) = 0.5$ and $p(A \cup B) = 0.55$. Calculate **(4 marks)**
the probabilities of the following events:

a. $p(A|B)$ **(2M)** b. $p(B'|A')$ **(2M)**

2. Simplify. Write "undefined" for expressions that are undefined. **(3 marks)**

a. $\begin{bmatrix} 3 & 1 & 3 \\ 0 & 5 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 6 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -6 & -1 \\ 1 & 1 & 4 \end{bmatrix}$ **(2M)** b. $\begin{bmatrix} 1 & -6 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix} - \begin{bmatrix} -3 \\ 0 \\ 3 \\ 2 \end{bmatrix}$ **(1M)**

3. In this question you will work determinants and row operations.

(4 marks)

Let $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ and $B = \begin{bmatrix} w & x \\ y + kw & z + kx \end{bmatrix}$.

- a. Find the determinant of matrix A. **(1M)** b. Find the determinant of matrix B. **(1M)**
- c. What row operation(s) were performed on matrix A in order to create the second row of matrix B? **(1M)** d. What conclusion can you make about the effect of row operations upon the determinant of a matrix? **(1M)**

4. A bag contains 16 marbles, 7 of which are red and the remaining 9 are white. On Quang's first selection, he selects a marble, records its color, and places it BACK IN THE BAG along with TWO OTHER MARBLES OF THE SAME COLOR. Quang then selects a marble and records its color.

(4 marks)

- a. Draw a tree diagram to show the sample space of outcomes and its probabilities. **(2M)** b. Find the probability that the two marbles that Quang selected are the same color. **(2M)**

5. A triangle is defined by the three points $A(2, 3)$, $B(-1, 2)$ and $C(3, -4)$. Use transformational matrices to find the coordinates of the image triangle if the triangle undergoes: **(5 marks)**
- a. a horizontal stretch by a factor of 3, **(3M)** followed by a vertical translation of 2 units upwards.
- b. a rotation of 135° counterclockwise about the origin **(2M)**

6. This question involves non-mutually exclusive events A , B , and C . **(4 marks)**
- a. Use a Venn diagram to explain why $p(A \cup B \cup C)$ is equal to **(2M)**
 $p(A) + p(B) + p(C) - p(A \cap C) - p(A \cap B) - p(B \cap C) + p(A \cap B \cap C)$

- b. Use the result from part (a) to find an expression for $p(A \cup B \cup C \cup D)$ for four non-mutually exclusive events A , B , C , and D . **(2M)**

7. A linear system is defined by the equations $px + 4y = 3k$ and $(p-3)x - 2y = 6$. (7 marks)

- a. Write a matrix equation to represent this linear system. (1M) b. Find the determinant of the coefficient matrix. (1M)

c. Hence or otherwise, determine the value(s) of p and k such that the linear system has:

- i. a unique solution (2M) ii. infinite solutions (2M) iii. no solutions (1M)

8. In this question you will work with the matrix $A = \begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$. (6 marks)

- a. Row reduce the augmented matrix (4M) b. What is the significance of the row reduced augmented matrix? Justify your conclusion. (2M)

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 4 & 1 & 0 & 0 \\ 2 & 3 & 2 & 0 & 1 & 0 \\ 0 & 5 & -1 & 0 & 0 & 1 \end{array} \right]$$

SECTION B: Calculator Active – 25 minutes

9. It is known that 4 out of every 1000 sophomores at ISM have a condition called Senioritis. A test has been devised that is 97% accurate in correctly determining whether a sophomore has the disease or not. **(4 marks)**
- a. Find the probability of a randomly selected sophomore getting a false positive on the test (meaning that the sophomore does not have Senioritis, but the test incorrectly determines that they do have it). **(2M)**
- b. Find the probability that a sophomore actually has Senioritis, given that they tested positive. **(2M)**
10. Mr. Santowski is playing golf and is watching his tee shot fly through the air (a PERFECT shot, of course). He notes the time and distance of his tee shot and records the following data relating the flight time (in t seconds) and the height of the ball (in h meters): (1, 19.5), (2, 19) and (3, 1.5) Mr Santowski knows that any projectile's height as a function of flight time can be modeled using the equation $h(t) = at^2 + bt + c$. **(4 marks)**
- a. Use the data points and the $h(t)$ equation to generate 3 equations involving the parameters a , b , and c . **(1M)**
- b. Write an augmented matrix that can be used to solve for the parameters a , b , and c . **(1M)**
- c. Hence or otherwise, determine the equation that can be used to model the ball's trajectory. **(2M)**

11. Solve the matrix equation $\begin{bmatrix} 24 & -4 \\ -16 & 12 \end{bmatrix} = \begin{bmatrix} 7 & -2 \\ 3 & -2 \end{bmatrix} \times X$ for the matrix X .

(2 marks)

12. Tim travels to ISM by bus 5 days a week, from Monday to Friday. The probability that he catches the 06:00 a.m. bus on Monday is 0.35. The probability that he catches the 06:00 a.m. bus on any other day is 0.75. A weekday is chosen at random.

(6 marks)

a. Are the events (day of the week and catching the bus) dependent or independent? Explain how you know.

(2M)

b. Find the probability that Tim catches the 06:00 a.m. bus on a randomly chosen weekday.

(2M)

c. Given that Tim catches the 06:00 a.m. bus on a randomly chosen weekday, find the probability that the chosen weekday is a Monday.

(2M)

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13. Alex scores on 85% of his free throws, while Bill scores on 75% of his free throws. Determine the probabilities of the following events: **(6 marks)**

- a. Alex misses his next free throw AND Bill scores on at least 1 of his next 4 free throws. **(3M)**
- b. Alex scores on exactly 2 of his next 3 free throws OR Bill misses exactly 1 of his next 2 free throws. **(3M)**

STUDENT SELF-EVALUATION

After the time allocated for writing this assessment has passed (or if you have finished early), answer the following questions:

- a. Estimate the letter grade that you achieved on this assessment (e.g. A-, C+, etc.): _____
- b. Which concepts did you have the most difficulty with during this assessment and/or this unit?

COMMUNICATION

In every formal assessment this year, 2 marks, 1 mark, or 0 marks will be awarded for the clarity of your communication in the presentation of your solutions and your written explanations.

On this assessment, you were awarded: _____ / 2 marks for communication.