MATH HONORS 2: SEMESTER 2 EXAM REVIEW

1) Simplify each expression. Write your result with no negative, fractional, or zero exponents.

(a) $\dfrac{(a^{2x})^3}{a^{-4x}}$ (b) $\sqrt{49a^8b^{-4}c^3}$ (c) $(x^{\frac{1}{2}} + y^{\frac{3}{2}})(x^{\frac{1}{2}} - y^{\frac{3}{2}})$

(d) $\dfrac{x^a\sqrt{x^{3a}}}{x^{-2a}}$ (e) $\left(\dfrac{3x^{-1}}{2a^2}\right)^{-2}\left(\dfrac{4x^2}{27a^{-3}}\right)^{-1}$

2) List the transformations needed to transform the graph of $f(x) = 2^x$ into the graph of $g(x) = 2^{x-5} - 3$.

3) Visitors to an island have increased by 6% per annum each year since the year 2000. If 4000 people visited the island in the year 2000, how many would be expected to visit the island in the year 2010?

4) Solve for x (by graphing) if $8^{2x-3} = 16^{2-x}$.

5) How much money will be in a savings account with an initial deposit of $2500 and an interest rate of 2.4% compounded monthly for 5 years?

6) A colony of 1000 weevils grows exponentially to 1750 in one week. How many weeks does it take for the weevil population to reach 3000?

7) Matrix $Q = \begin{bmatrix} -3 & 3 & 3 & -2 \\ 3 & 2 & -2 & -1 \end{bmatrix}$ represents the vertices of quadrilateral $ABCD$. Let matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Find the product PQ.

(b) Graph $A'B'C'D'$, the quadrilateral represented by PQ.

(c) Describe the transformation caused by matrix P.

8) Solve for X, the matrix equation $3AXB = 2C$.

9) If possible, multiply $\begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ -3 & -2 \end{bmatrix}$.

10) Find B if $A=\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}$ and $AB=\begin{bmatrix}5 & -9\\ 4 & -3\end{bmatrix}$.

11) Given that $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$,

(a) Find $|A|$ (or "det A" or "the determinant of A").

(b) If possible, find the inverse of A.

(c) A singular matrix means that _____.

Therefore, A is singular/non-singular. [Circle the correct choice.]

12) State the conditions for m, n, p, q if we can evaluate $2A-3B$ where A is a matrix of order $m\times n$ and B is a matrix of order $p\times q$.

13) Find the value(s) of a for which the system of equations $\begin{aligned}3x-ay+2z&=4\\ x+2y-3z&=1\\ -x-y+z&=12\end{aligned}$ has a unique solution.

14) Solve for x:

(a) $\log_6 x=-2$

(b) $2\log_a x+\log_a 4=\log_a(4x+3)$

(c) $\log_7(\log x)=0$

(d) $\log_7 x=\frac{1}{3}\log_7 64+\frac{1}{2}\log_7 121$

15) The remaining amount of a 100 milligram dose of a painkiller in a person's bloodstream after t hours can be modeled by the function $A(t)=100e^{-0.21t}$. Find the remaining amount of the painkiller after 4 hours.

16) Solve for x, writing your solution as an expression: $4e^{3x-5}-2=70$

17) Find a negative angle that is co-terminal to $468°$, expressing your answer in radians.

18) Evaluate $\arcsin\left(-\sqrt{2}\sin\frac{\pi}{6}\right)$

19) Solve for x:

(a) $\sqrt{2}\csc 2x + 2 = 0$, where $0 \le x \le \pi$

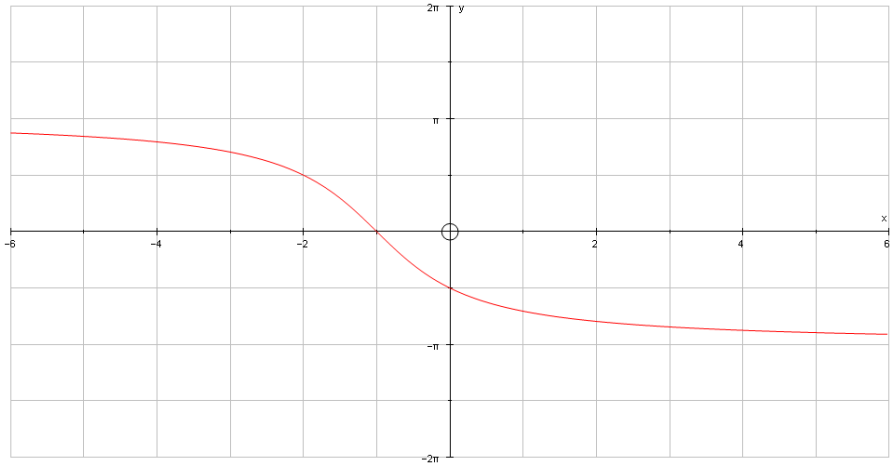
(b) $\sqrt{3}\tan x + 1 = \sec^2 x + \tan x - \sqrt{3}$, where $-\pi \le x \le \pi$

20) Graph the function $y = 2\arccos\left(-x + \frac{1}{2}\right) - \pi$, labeling any intercepts and endpoints with coordinates.

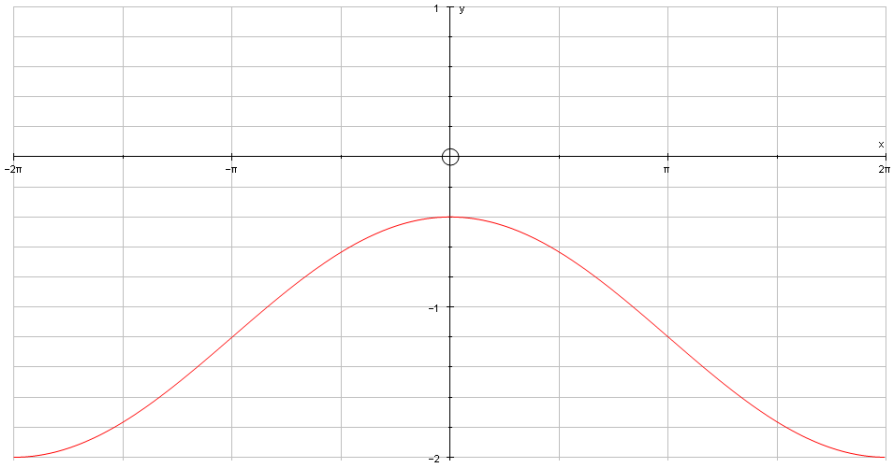
21) Prove that $\cos\theta + \tan\theta \sin\theta = \sec\theta$

22) Prove that $\frac{\csc x - \cot x}{\sec x - 1} = \cot\theta$

23) Find the values of A and C given the graph of the function $y = A\arctan(x + C)$:



24) Find the values of A, B, and D given the graph of the function $y = A\cos Bx + D$:



25) Given that $\tan\theta = \frac{2}{3}$ where θ is an acute angle, find the exact value of $\cot(\theta + 30°)$.

26) Find an expression for the exact value of $\cos\frac{\pi}{12} - \sin\frac{\pi}{8}$.

27) A boy is lying on a beach, looking at a mountain that is 1800 meters tall. If the distance between the boy and the top of the mountain is 20 kilometers, find the angle of elevation from the boy to the top of the mountain.

28) A football is kicked in the air above the middle of a football field. The angle of elevation from one of the goalies to the football is $38.6°$, the angle of elevation from the other goalie to the football is $29.1°$, and the two goals are 100 yards apart. What is the distance between the football and the goalie nearest to the football?

29) ΔABC has side lengths $a = 12.3$ km, $b = 8.4$ km and $c = 14.9$ km

(a) Find the measure of the largest angle in ΔABC

(b) Find the area of ΔABC

30) A café has 7 kinds of alcoholic beverages and 5 kinds of hot desserts on the adult menu, and it has 4 kinds of non-alcoholic beverages and 8 kinds of cold desserts on the children's menu. The café is running a "Snack Pack" promotion that allows an adult or a child to get a beverage **and** a dessert for just 100 pesos, although both items must come from the same menu. In how many ways can a "Snack Pack" be ordered?

31) A coin is flipped 4 times. Find the probability of getting exactly 2 heads.

32) If a car drives through a green traffic light, then it has a 60% probability of being stopped by a red traffic light at the next intersection. If a car is waiting at a red traffic light, then there is a 75% probability that the next intersection it reaches will have a green traffic light. Suppose that a car is waiting at a red traffic light. What is the probability that the car will get stopped at a red traffic light two intersections later?