2 Find the equation of the line through:

- **a** A(8, 4) and B(5, 1)
- **b** A(5, −1) and B(4, 0)
- **c** A(−2, 4) and B(−3, −2)
- **d** P(0, 6) and Q(1, −3)
- **e** M(−1, −2) and N(5, −4)
- **f** R(−1, −4) and S(−3, 2).

3 Find the equation of the line:

- **a** which has gradient $-\frac{1}{2}$ and cuts the y-axis at 4
- **b** which is parallel to a line with slope 3, and passes through the point (−1, −2)
- **c** which cuts the x-axis at 4 and the y-axis at 3
- **d** which cuts the x-axis at −2, and passes through (2, −5)
- **e** which is perpendicular to a line with gradient $\frac{1}{2}$, and cuts the x-axis at −1
- **f** which is perpendicular to a line with gradient −3, and passes through (4, −1).

H THE GENERAL FORM OF A LINE

Consider the line $y = -\frac{2}{3}x + \frac{11}{3}$. Its equation is given in gradient-intercept form.

Equations in this form often contain fractions. We can remove them as follows:

If $y = -\frac{2}{3}x + \frac{11}{3}$ then

$3y = -2x + 11$ {multiplying each term by 3}

$\therefore \quad 2x + 3y = 11$

The equation is now in the form $Ax + By = C$, where $A = 2$, $B = 3$, $C = 11$.

> $\boldsymbol{Ax + By = C}$ is called the **general form** of the equation of a line.
> A, B and C are constants, and x and y are variables.

Example 14 Self Tutor

Find, in *general form*, the equation of a line:

- **a** through (2, 5) with slope $-\frac{1}{3}$
- **b** through (1, 3) and (2, −1).

a The equation is

$$\frac{y-5}{x-2} = \frac{-1}{3}$$

$\therefore \quad 3(y-5) = -1(x-2)$

$\therefore \quad 3y - 15 = -x + 2$

$\therefore \quad x + 3y = 17$

b the gradient $= \dfrac{-1-3}{2-1} = -4$

$\therefore$ the equation is

$$\frac{y-3}{x-1} = -4$$

$\therefore \quad y - 3 = -4(x-1)$

$\therefore \quad y - 3 = -4x + 4$

$\therefore \quad 4x + y = 7$

When the equation of a line is given in the general form, we can rearrange it to the form $y = mx + c$ so that we can determine its gradient.

Example 15 — Self Tutor

Find the gradient of the line $3x + 4y = 10$.

$3x + 4y = 10$

$\therefore \quad 4y = -3x + 10$ {subtracting $3x$ from both sides}

$\therefore \quad y = -\dfrac{3x}{4} + \dfrac{10}{4}$ {dividing each term by 4}

$\therefore \quad y = -\frac{3}{4}x + \frac{5}{2}$ and so the slope is $-\frac{3}{4}$

EXERCISE 6H.1

1 Find, in *general form*, the equation of the line through:

- **a** $(1, 4)$ having gradient $\frac{1}{3}$
- **b** $(-2, 1)$ having gradient $\frac{3}{5}$
- **c** $(6, 0)$ having gradient $-\frac{2}{3}$
- **d** $(4, -1)$ having gradient $\frac{4}{5}$
- **e** $(-4, -2)$ having gradient 3
- **f** $(3, -1)$ having gradient -2.

2 Find, in *general form*, the equation of the line through:

- **a** A$(8, 4)$ and B$(5, 1)$
- **b** C$(5, -1)$ and D$(4, 0)$
- **c** E$(-2, 4)$ and F$(-2, -3)$
- **d** G$(1, -3)$ and H$(0, 6)$
- **e** I$(-2, -1)$ and J$(-1, 2)$
- **f** K$(-1, -4)$ and L$(-2, -3)$.

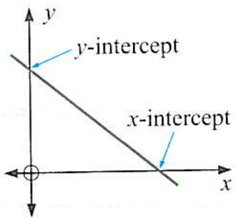
3 Find the gradient of the line with equation:

- **a** $y = 2x + 3$
- **b** $y = 2$
- **c** $y = -3x + 2$
- **d** $x = 5$
- **e** $y = 2 - 4x$
- **f** $y = 3 + \frac{2}{3}x$
- **g** $y = \dfrac{3x + 1}{4}$
- **h** $y = \dfrac{2 - 3x}{5}$
- **i** $3x + y = 4$
- **j** $2x + 3y = 8$
- **k** $3x + 5y = 11$
- **l** $4x + 7y = 20$
- **m** $x - 2y = 4$
- **n** $3x - 4y = 12$
- **o** $5x - 6y = 30$

GRAPHING FROM THE GENERAL FORM

The easiest method used to graph lines in the general form $Ax + By = C$ is to use axes intercepts.

The x-intercept is found by letting $y = 0$.
The y-intercept is found by letting $x = 0$.



Example 16 **Self Tutor**

Graph the line with equation $4x - 3y = 12$ using axes intercepts.

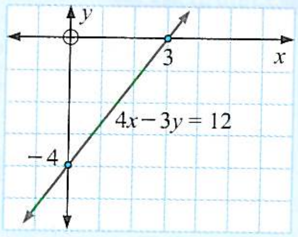
For $4x - 3y = 12$

when $x = 0$, $-3y = 12$

$\therefore\ y = -4$

when $y = 0$, $4x = 12$

$\therefore\ x = 3$



EXERCISE 6H.2

1 Use axes intercepts to sketch graphs of:

a $x + 3y = 6$ **b** $3x - 2y = 12$ **c** $2x + 5y = 10$

d $4x + 3y = 6$ **e** $x + y = 5$ **f** $x - y = -3$

g $3x - y = -6$ **h** $7x + 2y = 14$ **i** $3x - 4y = -12$

I POINTS ON LINES

A point lies on a line if its coordinates satisfy the equation of the line.

For example:

(2, 3) lies on the line $3x + 4y = 18$ since $3 \times 2 + 4 \times 3 = 6 + 12 = 18$ ✓

(4, 1) does not lie on the line since $3 \times 4 + 4 \times 1 = 12 + 4 = 16$.

EXERCISE 6I

1 **a** Does (3, 4) lie on the line with equation $5x + 2y = 23$?

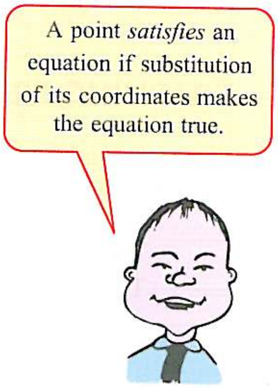
b Does (−1, 4) lie on the line with equation $3x - 2y = 11$?

c Does $(5, -\frac{1}{2})$ lie on the line with equation $3x + 8y = 11$?

2 Find k if:

a (2, 5) lies on the line with equation $3x - 2y = k$

b (−1, 3) lies on the line with equation $5x + 2y = k$.



3 Find a given that:

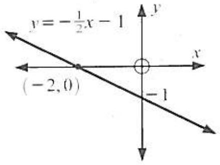
a $(a, 3)$ lies on the line with equation $y = 2x - 11$

b $(a, -5)$ lies on the line with equation $y = 4 - x$

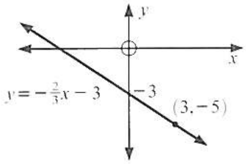
c $(4, a)$ lies on the line with equation $y = \frac{1}{2}x + 3$

d $(-2, a)$ lies on the line with equation $y = 1 - 3x$.

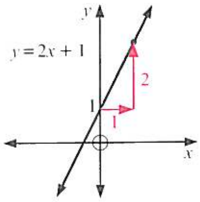
g



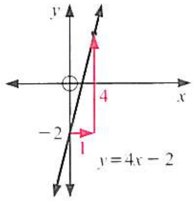
h



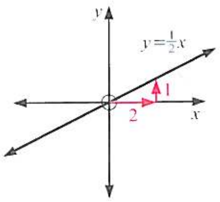
2 **a**



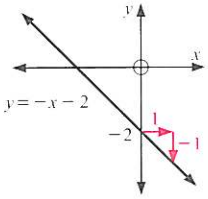
b



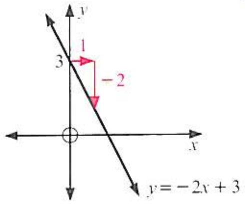
c



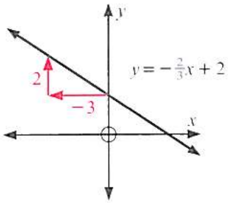
d



e



f



EXERCISE 6G.3

1 **a** $y=2x+1$ **b** $y=-x+1$ **c** $y=-3x+10$
d $y=\frac{2}{3}x+\frac{7}{3}$ **e** $y=-\frac{1}{2}x-\frac{1}{2}$ **f** $y=-3$
g $y=\frac{2}{3}x+\frac{17}{3}$ **h** $y=-\frac{4}{5}x+\frac{6}{5}$ **i** $y=-\frac{3}{4}x+\frac{3}{2}$
j $y=\frac{4}{7}x-\frac{26}{7}$

2 **a** $y=x-4$ **b** $y=-x+4$ **c** $y=6x+16$
d $y=-9x+6$ **e** $y=-\frac{1}{3}x-\frac{7}{3}$ **f** $y=-3x-7$

3 **a** $y=-\frac{1}{2}x+4$ **b** $y=3x+1$ **c** $y=-\frac{3}{4}x+3$
d $y=-\frac{5}{4}x-\frac{5}{2}$ **e** $y=-2x-2$ **f** $y=\frac{1}{3}x-\frac{7}{3}$

EXERCISE 6H.1

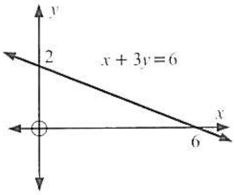
1 **a** $x-3y=-11$ **b** $3x-5y=-11$
c $2x+3y=12$ **d** $4x-5y=21$
e $3x-y=-10$ **f** $2x+y=5$

2 **a** $x-y=4$ **b** $x+y=4$ **c** $x=-2$
d $9x+y=6$ **e** $3x-y=-5$ **f** $x+y=-5$

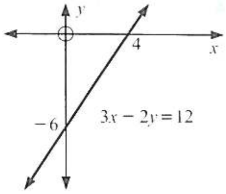
3 **a** 2 **b** 0 **c** -3 **d** undefined **e** -4
f $\frac{2}{3}$ **g** $\frac{3}{4}$ **h** $-\frac{3}{5}$ **i** -3 **j** $-\frac{2}{3}$ **k** $-\frac{3}{5}$
l $-\frac{4}{7}$ **m** $\frac{1}{2}$ **n** $\frac{3}{4}$ **o** $\frac{5}{6}$

EXERCISE 6H.2

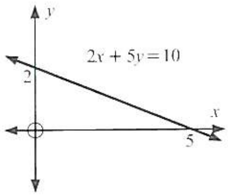
1 **a**



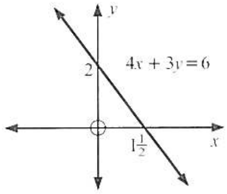
b



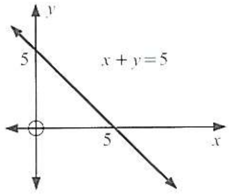
c



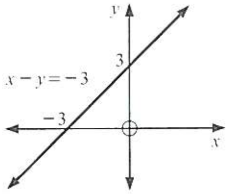
d



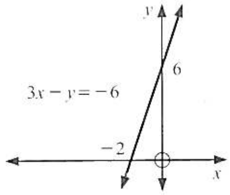
e



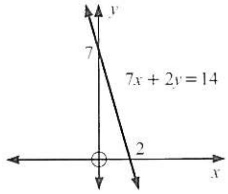
f



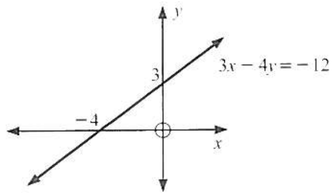
g



h



i



EXERCISE 6I

1 **a** yes **b** no **c** yes **2** **a** $k=-4$ **b** $k=1$
3 **a** $a=7$ **b** $a=9$ **c** $a=5$ **d** $a=7$
4 **a** $b=-3$ **b** $b=-\frac{9}{4}$ **c** $b=-\frac{7}{5}$ **d** $b=\frac{5}{4}$
e $b=3$ **f** $b=\frac{2}{3}$

EXERCISE 6J

1 **a** $(-1,2)$ **b** $(2,4)$ **c** $(3,1)$ **d** $(-2,3)$ **e** $(0,6)$
f $(-1,2)$ **g** $(2,1)$ **h** no point of intersection
i infinitely many points of intersection

2 **a** None, as the lines are parallel.
b Infinitely many, as the lines are coincident.
c If $k=5$, infinitely many, as the lines are coincident;
if $k\neq 5$, none, as the lines are parallel.

REVIEW SET 6A

1 **a** $2\sqrt{17}$ units **b** $(2,2)$ **c** -4 **d** $C(-1,14)$
2 $a=-2$ or 4
3 $PQ=PR=\sqrt{20}$ units, $QR=4$ units, isosceles triangle