T1.1 – Sequences & Series
Lesson 2 - Geometric Sequences

Math SL1 - Santowski

FAST FIVE

- Comment upon any pattern you see in the sequences
- ex 1. 2,10,50,250,.....
- ex 2. 5,-10,20,-40,80,.....
- ex 3. 3,6,12,24,48,....
- ex 4. 2,4,8,16,32,64,....
- ex 5. 100, 50, 25, 12.5, 6.25, …
- Now PREDICT a formula that you can use to tell me the 15^{th} term of each sequence

T1.1 - Lesson 2 Objectives

- The student will predict patterns in arithmetic & geometric sequences and then write algebraic expressions for these patterns
- The student will apply arithmetic and geometric sequences to financial models

(A) Review

- A sequence is an ordered set of numbers.
- An arithmetic sequence has a pattern to it ➔ the constant difference between successive terms.
- We have developed a formula (called the general term) that we can use to determine any term of the sequence $u_n = u_1 + (n-1)d$
- Today, we will explore other sequences that have another pattern to them.

(B) The General Term of a Geometric Sequences

- Consider the following analysis:
- $t_1 = 3 = 3 \times 1 = 3 \times 2^0$
- $t_2 = 6 = 3 \times 2 = 3 \times 2^1$
- $t_3 = 12 = 3 \times 4 = 3 \times 2 \times 2 = 3 \times 2^2$
- $t_4 = 24 = 3 \times 8 = 3 \times 2 \times 2 \times 2 = 3 \times 2^3$
- $t_5 = 48 = 3 \times 16 = 3 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^4$
- We can see a pattern emerging as to how to calculate the general term of a geometric sequence as ???????

(B) The General Term of a Geometric Sequences

- To calculate the general term of a geometric sequence as:
- $u_n = ar^{n-1}$, where a is the first term of the sequence, n is the term number, and r is the common ratio.
- Working with the formula $u_n = ar^{n-1}$, we notice two things:
- If $r > 1$, then the terms increase
- Ex: 2,10,50,250,.....
- If $0 < r < 1$, then the terms decrease
- Ex: 100, 50, 25, 12.5, 6.25, …

(C) Examples – GDC and non GDC solutions

- ex 1. Write the first 6 terms of the sequence defined by $u_n = 5(-2)^{n-1}$
- ex 2. Given the formula for the nth term as $u_n = -5(4)^{n-1}$, find 10^{th} term.
- ex 3. Find the formula for the *n*th term given the geometric sequence 2,6,18,…… Then find the 7^{th} term.
- ex 4. How many terms are there in the geometric sequence 3,6,12,….,384
- ex 5. If the 5^{th} term of a sequence is 1875 and the 7^{th} term is 46,875, find *a*, *r*, and u_n and the first three terms of the sequence.

(C) Examples – GDC solution

- ex 6. Since 1967, the average annual baseball salary was $19,000. The average annual salary has been rising at a rate of 17% of the previous year's salary. Determine the equation for this sequence and then predict the average annual salary for 2007.
- Ex 7. Since 1967, the average annual soccer salary was $19,000. The average annual salary has been rising at a rate of 17% of the 1967 salary. Determine the equation for this sequence and then predict the average annual salary for 2007.
- Graph the two sequences from Ex 6 & 7 and compare and contrast the salaries
- ex 8. The half life of iodine-131 is 8 days. What amount will remain in 112 days if you started with 12 mg of iodine-131? Determine the equation for geometric sequence.

(D) Financial Applications: Definitions

- **Simple interest** means that only the principal invested earns interest
- **Principal** means the original amount invested or borrowed
- **Rate** refers to the interest rate being paid as a percent usually per year (per annum)
- The **amount of the investment** refers to the total of the principal and the interest being paid

(D) Financial Applications: Formula for Simple Interest

- $I = Prt$ where *I* is the interest earned, *P* is the principal invested, *r* is the interest rate being earned (usually per annum), and *t* is the length of time that the principal earns interest (usually in years)
- $A = P + I$ where *A* is the total amount of the investment which is the sum of the original principal and the interest earned
- upon combining $A = P(1 + rt) = P + Prt$
- Q? does this equation look familiar?????

(D) Financial Applications: Examples with Simple Interest

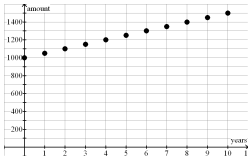
I = ?	P=$2500	r=3.75%/a	T = 3a	A = ?
I = ?	P=$1200	r=4.25%/a	T = 6m	A = ?
I = ?	P=$3000	r=3%/a	T = ?	A = $3120
I = ?	P=$2000	R = ?	T = 1.5a	A = $2120
I = ?	P = ?	r=4%/a	T = 2a	A = $4050
I = ?	P = ?	r=4.25%/a	T = 175d	A = $1200

(D) Financial Applications: Arithmetic Sequences

- If you invest $1000 for 10 years which earns 5% pa simple interest:
- (I) Determine the value of the total investment at the end of each year for the next 10 years.
- (ii) List the terms and determine the general term. What type of sequence is this?
- (iii) Graph the sequence with an appropriate choice of axis.
- (iv) Explain why simple interest is an example of "linear growth"

(D) Financial Applications: Arithmetic Sequences

- if you list the investment amount yearly, you get the sequence (1000?), 1050, 1100, 1150, 1200,...1500 which we can write as:
- $u_n = a + (n - 1)d$ or
- $u_n = 1050 + (n - 1)50$ or
- $u_n = 50n + 1000$



13

(E) Financial Applications: Compound Interest

- **Compound Interest** means that you earn interest on the original investment plus any previously accrued interest i.e. you interest on your principal and also on your interest
- **Compounding period** refers to the interval of time that interest is accrued prior to being added into the principal i.e. your interest is added into your principal at the end of the compounding period
- ex. if you invest $4000 at 10% pa compounded annually, it means that interest earned is calculated every year and added into the principal at the end of every year.

14 8/7/2009

(E) Financial Applications: Compound Interest

- Compounding Periods:
- **Semi-annual** => interest determined and added every 6 months, twice a year
- **Quarterly** => interest determined and added every 3 months, 4 times a year
- **Monthly** => interest determined and added every month, 12 times a year
- **Daily** => interest determined and added every day, 365 times a year

15 8/7/2009

(E) Financial Applications: Compound Interest

- ex. if you invest $4000 at 10% pa compounded annually, determine how much you have at the end of 10 years?
- $4400, $4840, $5324, $5856.4, etc....
- Rather than repeating the simple interest calculation formula over and over again, we have a simple formula to use:
- $A = P(1 + i)^n$
- A = total amount of investment
- P = the principle investment
- i = interest rate per conversion period
- n = number of conversion periods

16 8/7/2009

(E) Financial Applications: Compound Interest

- Determine the total value of your investment if the conditions of your investment are:
- P=$2000 @ 3% quarterly for 2a
- P=$1500 @ 4% semi-annually for 4 a
- P=$500 @ 1% daily for 275 d
- P=$1250 @ 2.4% monthly for 1½ a

17 8/7/2009

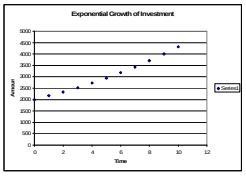
(E) Financial Applications: Compound Interest Compound Interest as a Sequence

- If you invest $2000 for 6 years at 8%pa compounded annually,
- (i) Determine the value of the total investment at the end of each year for the next 10 years.
- (ii) List the terms and determine the general term
- (iii) Graph the sequence with an appropriate choice of axis.

18 8/7/2009

(E) Financial Applications: Compound Interest
Compound Interest as a Sequence

- if you list the investment amount yearly, you get a sequence which we can write as $u_n = ar^{n-1}$
- or $u_n = 2000(1+0.08)^{n-1}$



(E) Homework

- Ex 2D #2, 6c, 7a, 8c, 9, 15, 19;
- Ex 4F#1,2