T5.3 – Vectors & Motion

IB Math SL1 - Santowski

(A) Review – Basic Physics

- The velocity (or speed) of an object is defined as the rate at which the object's position changes
- Speed of an object is a scalar quantity (magnitude or amount of speed only)
- Velocity of an object is a vector quantity (magnitude as well as a direction)

(A) Review – Basic Physics

- A basic physics formula for speed is distance/time and so a basic formula for distance is speed x time

$$speed = \frac{\text{distance}}{\text{time}} \qquad \text{distance} = speed \times time$$
$$s = \frac{d}{t} \qquad d = st$$

- A basic physics formula for velocity is displacement/time and a basic formula for displacement is velocity x time

$$velocity = \frac{\text{displacement}}{\text{time}} \qquad displacement = velocity \times time$$
$$\vec{v} = \frac{\vec{d}}{t} \qquad \vec{d} = \vec{v} \times t$$

(A) Review – Basic Physics

- Now let's take our basic physics formula for velocity is displacement/time and a basic formula for displacement is velocity x time and introduce a slight change:
- How do I determine the displacement of an object?
- I would find the difference between its initial position and its final position:
- displacement = change in position = $\Delta d = d_f - d_i$

$$velocity = \frac{\Delta d}{time} = \frac{d_f - d_i}{t} \qquad \Delta d = d_f - d_i = velocity \times time$$
$$\vec{v} = \frac{\vec{d}}{t} = \frac{\Delta \vec{d}}{time} = \frac{\vec{d}_f - \vec{d}_i}{t} \qquad \vec{d} = \Delta\vec{d} = \vec{d}_f - \vec{d}_i = \vec{v} \times t$$

(A) Review – Basic Physics

- And now we will make one final adjustment to our formula:
- displacement = change in position = $\Delta d = d_f - d_i = v\,t$

$$\vec{d}_f - \vec{d}_i = \vec{v} \times t$$
$$\vec{d}_f = \vec{d}_i + \vec{v} \times t$$
$$\vec{d}_f = \vec{d}_i + t \times \vec{v}$$

- So does the equation $\boldsymbol{d}_f = \boldsymbol{d}_i + t\boldsymbol{v}$ look familiar?

(A) Review – Basic Physics

- And now we will make one final adjustment to our formula:
- displacement = change in position = $\Delta d = d_f - d_i = v\,t$

$$\vec{d}_f - \vec{d}_i = \vec{v} \times t$$
$$\vec{d}_f = \vec{d}_i + \vec{v} \times t$$
$$\vec{d}_f = \vec{d}_i + t \times \vec{v}$$

- So does the equation $\boldsymbol{d}_f = \boldsymbol{d}_i + t\boldsymbol{v}$ look familiar?
- Consider $\boldsymbol{b} = \boldsymbol{a} + t\boldsymbol{d}$
- Consider $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

(A) Review – Basic Physics

- So does the equation $\boldsymbol{d}_f = \boldsymbol{d}_i + t\boldsymbol{v}$ look familiar?
- Consider $\boldsymbol{b} = \boldsymbol{a} + t\boldsymbol{d}$ or consider $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$
- So if we consider the equation $\boldsymbol{b} = \boldsymbol{a} + t\boldsymbol{d}$, **in the new context of motion**, what do the parameters and variables represent??
- b
- a
- t
- d

(A) Review – Basic Physics

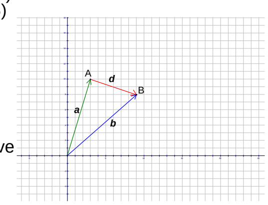
- So does the equation $\boldsymbol{d}_f = \boldsymbol{d}_i + t\boldsymbol{v}$ look familiar?
- Consider $\boldsymbol{b} = \boldsymbol{a} + t\boldsymbol{d}$ or consider $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$
- So if we consider the equation $\boldsymbol{b} = \boldsymbol{a} + t\boldsymbol{d}$, **in the new context of motion**, what do the parameters and variables represent??
- b ➔ position vector of the object at some arbitrary time ("final" position)
- a ➔ position vector of the object at the "beginning"
- t ➔ the time elapsed between position 1 (beginning) and position 2 (end)
- d ➔ velocity vector

(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- To study the vector equations and motion, let's define 2 points as: A(3,10) and B(9,8) ➔ but now let's consider them as POSITIONS ON A MAP AS WE MOVE from position A to position B
- And to simply illustrate our point, let's further add the element of time ➔ our change in positions from A to B took one hour

(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- Our "starting" point was A(3,10) and our "final" point was B(9,8)
- Draw the line segment AB
- AB= [6,-2]
- If we consider vector $\boldsymbol{d}$ ➔ we travelled 6 km in a positive x direction and 2 km in a negative direction in one hour
- Recall that d = vt
- So that d = 1 x [6,-2]

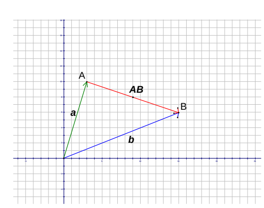


(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- To continue our study the vector equation and motion, let's define 2 points as: A(3,10) and B(15,6) ➔ and again let's consider them as POSITIONS ON A MAP AS WE MOVE from position A to position B
- And to simply illustrate our point, let's further add the element of time ➔ our change in positions from A to B took another one hour for a total elapsed time of 2 hours

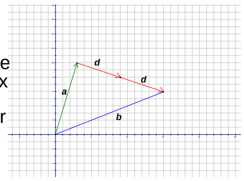
(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- Our "starting" point was A(3,10) and our "final" point was B(15,6)
- Draw the line segment AB
- AB= [12,-4]
- If we consider vector $\boldsymbol{d}$ ➔ we travelled 12 km in a positive x direction and 4 km in a negative direction in two hour
- Recall that d = vt,
- so that [12,-4] = 2 x [6,-2]



(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- Our “starting” point was A(3,10) and our “final” point was B(15,6)
- If we consider vector ***d*** ➔ we travelled 12 km in a positive x direction and 4 km in a negative direction in two hour



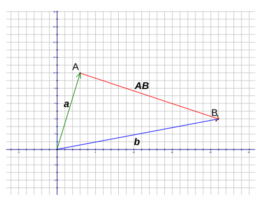
- Recall that d = vt,
- so that [12,-4] = 2 x [6,-2]

(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- To continue our study the vector equation and motion, let’s define 2 points as: A(3,10) and B(21,4) ➔ and again let’s consider them as POSITIONS ON A MAP AS WE MOVE from position A to position B
- And to simply illustrate our point, let’s further add the element of time ➔ our change in positions from A to B took another one hour for a total elapsed time of 3 hours

(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

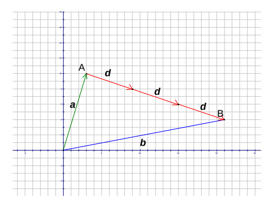
- Our “starting” point was A(3,10) and our “final” point was B(21,4)
- Draw the line segment AB
- AB= [18,-6]
- If we consider vector ***d*** ➔ we travelled 18 km in a positive x direction and 6 km in a negative direction in three hours



- Recall that d = vt,
- so that [18,-6] = 3 x [6,-2]

(B) ALTERNATE APPROACH ➔ Vector Equations of Lines

- Our “starting” point was A(3,10) and our “final” point was B(21,4)
- If we consider vector ***d*** ➔ we travelled 18 km in a positive x direction and 6 km in a negative direction in three hours



- Recall that d = vt,
- so that [18,-6] = 3 x [6,-2]

(C) Vector Equations of Lines & Motion - Summary

- So now our vector equation ***b*** = ***a*** + t***d*** has new meanings for ***b***,***a***,t,***d*** that are consistent with the motion application:
- b ➔ position vector of the object at some arbitrary time (“final” position)
- a ➔ position vector of the object at the “beginning”
- t ➔ the time elapsed between position 1 (beginning) and position 2 (end)
- d ➔ velocity vector
- And given our new context, I will rewrite the vector equation for motion as $\mathbf{b} = \mathbf{a} + t\mathbf{v}$ or $\mathbf{d}_f = \mathbf{d}_i + t\mathbf{v}$

(D) Examples

- The vector equation of a moving object is given by [x,y] = [7,5] + *t* [6,-8], where *t* is time in hours and distances are in km.
- (a) Find the initial position of the object.
- (b) Find the velocity of the object. EXPLAIN what the velocity vector means about the motion of the object.
- (c) Determine the speed of the object
- (d) Where is the object located 4 hours after it starts its motion

Example

- A racing car is moving with a velocity vector parallel to [-8,15] with a speed of 136 miles per hour. Determine the velocity vector of the racing car.

Example

- My son is riding his bike. His starting point is described by the position vector 5***i*** + 10***j*** m and he is moving with a velocity vector of 3***i*** – ***j*** m/min.
- (a) Determine his position at ANY point in time.
- (b) Determine his position 4 minutes after he started riding from his starting point
- (c) Determine his speed
- (d) If our house location is described by the position vector *x**i*** – 2***j***, at what time does he get home?
- (e) What is the value of *x* given our position vector *x**i*** – 2***j***

Example

- The parametric equations for a moving object are given by x(t) = 4 + 5t and y(t) = 5 – 2t, where t $\geq$ 0 seconds and x & y are measured in meters
- (a) Determine the vector equation describing the objects displacement.
- (b) What is the speed of the object?
- (c) An obstacle is located at the position described by the point (39,-9). Will the object collide with the obstacle?
- (d) Use your calculator to graph the objects motion.
- (e) EXPLAIN what you must do to determine the closest distance between the object to the origin

Example

- Andrew is riding his skateboard and Ian is riding on his scooter. Andrew's position at any moment is given by A = [-8,3] + t[4,3] and Ian's position at any moment is given by I = [2,-7] + t[2,5], where t $\geq$ 0 and is measured in minutes and all distances are in meters. Both boys start their motions at the same time.
- (a) Who has a greater speed?
- (b) How far apart are the boys 4 minutes after they begin riding?
- (c) If they maintain their motions as described, will Mr S have to get out the first aid kit?

(G) Homework

- HW
- Ex 17B #1bc,2ab, 3ad;
- Ex 17C #1, 2ad, 3, 4abe,
- IB Packet #2, 3