T5.3 – Lines in Space

IB Math SL1 - Santowski

Review – Equations of Lines in 2-Space

- The line y = mx + b can be written as a vector equation in the form of $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \end{bmatrix} + t\begin{bmatrix} d_x \\ d_y \end{bmatrix}$
- The line y = mx + b can be written as a parametric equation in the form of $x(t) = x_o + d_x t$, $y(t) = y_o + d_y t$
- The line y = mx + b can be written as a symmetrical equation in the form of $t = \frac{x_0 - x}{d_x} = \frac{y_0 - y}{d_y}$

Review – Equations of Lines in 3-Space

- The line y = mx + b can be written as a vector equation in the form of $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_o \\ y_o \\ z_0 \end{bmatrix} + t\begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$
- The line y = mx + b can be written as a parametric equation in the form of $x(t) = x_o + d_x t$, $y(t) = y_o + d_y t$, $z(t) = z_o + d_z t$
- The line y = mx + b can be written as a symmetrical equation in the form of $t = \frac{x_0 - x}{d_x} = \frac{y_0 - y}{d_y} = \frac{z_0 - z}{d_z}$

(A) Classifying Lines in 3-Space

- A pair of lines in 2-space can:
- (a) intersect if their slopes (direction vectors) are different
- (b) be parallel if their slopes (direction vectors) are equal
- (c) be coincident if their slopes and y-intercepts (or points) are identical
- What about lines in 3-space??

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- (a) intersect if their slopes (direction vectors) are different
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- (c) be coincident if their slopes and y-intercepts (or points) are identical
- (d) BUT also they can have different slopes (direction vectors) and NOT intersect ➔ HOW is this possible??
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(B) Skew Lines

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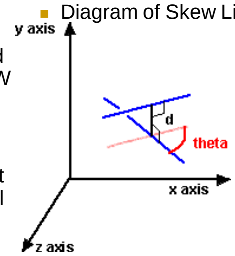
- Diagram of Skew Lines



(A) Classifying Lines in 3-Space

- If you are given 2 lines in 3-space, explain how you would:
- (a) determine that they were (or were not) parallel?
- (b) determine where (if) they intersect?
- (c) determine that (if) they were skew lines?

Example

Find the parametric equations of the line passing through the point P = (1, 3, –2) and parallel to the line with the vector equation **r** = [2t, 1+ t, 4 – 3t]

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Answer: We put the vector equation in the form:
r = **OP** + t**v** = [0, 1, 4] + t[2, 1, – 3],
thus getting the direction vector **v** = [2, 1, – 3].
Finally, we write the parametric equations:
x = 1 + 2t, y = 3 + t, z = –2 – 3t.

Example

Ex. Consider the line that passes through (5,1,3) and is parallel to **i** + 4**j** – 2**k**.

a) Find the vector and parametric equations of the line

b) Identify two other points on the line.

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(F) Examples

1 The points A and B have position vectors $\begin{pmatrix}1\\6\\4\end{pmatrix}$ and $\begin{pmatrix}5\\0\\-6\end{pmatrix}$ respectively, relative to a fixed origin.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line m has equation

$$\mathbf{r} = \begin{pmatrix}5\\-5\\3\end{pmatrix} + t\begin{pmatrix}1\\-4\\2\end{pmatrix}.$$

Given that lines l and m intersect at the point C,

b find the position vector of C, **(4)**

c show that C is the mid-point of AB. **(2)**

(F) Examples

2 Relative to a fixed origin, the points P and Q have position vectors $\begin{pmatrix}5\\-2\\2\end{pmatrix}$ and $\begin{pmatrix}3\\1\\0\end{pmatrix}$ respectively.

a Find, in vector form, an equation of the line L_1 which passes through P and Q. **(2)**

The line L_2 has equation

$$\mathbf{r} = \begin{pmatrix}4\\6\\-1\end{pmatrix} + \mu\begin{pmatrix}5\\-1\\3\end{pmatrix}.$$

b Show that lines L_1 and L_2 intersect and find the position vector of their point of intersection. **(5)**

c Find, in degrees to 1 decimal place, the acute angle between lines L_1 and L_2. **(3)**

Examples

- (a) Determine the measure of the acute angle between the lines

$$L_1: \frac{x-3}{2} = \frac{y+2}{-1} = \frac{z}{\sqrt{3}}$$

$$L_2: \frac{x+1}{1} = y-2 = \frac{z-1}{\sqrt{3}}$$

- (b) Line L passes through the points A(1,2,-1) and B(11,-2,-7), while line M passes through the points C(2,-1,-3) and D(9,-10,3). Classify the lines as (i) intersecting, (ii) parallel, (iii) coincident, (iv) skew

Examples

- (c) Line L passes through the points (4,3,9) and (7,8,5) while line M passes through the points (12,16,4) and (*k*,26,-4), where k is a real number. Find the value of k if:
  - (i) Line L is parallel to line M
  - (ii) Line L is perpendicular to line M

Examples

- (d) Find the co-ordinates of the point where the line L_1 intersects the x-y plane

$$L_1: \quad r = \begin{pmatrix}-2\\5\\3\end{pmatrix} + t\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

- (e) Find the values of a,b if the line L_2 passes through the point (a,1,b)

$$L_2: \quad \frac{x-3}{4} = y+2 = \frac{4-z}{5}$$

Example

- (f) Find the intersection point of the lines

$$L_1: \quad \frac{x-5}{-2} = y-10 = \frac{z+9}{12}$$

$$L_2: \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\9\\-9\end{bmatrix} + t\begin{bmatrix}0\\-2\\6\end{bmatrix}$$

- (g) Show that the following lines are skew

$$L_1: \frac{x-1}{-3} = y-2 = \frac{7-z}{11}$$

$$L_2: \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\-1\\4\end{pmatrix} + t\begin{pmatrix}3\\8\\-7\end{pmatrix}$$

Homework

- HW
- Ex 17F #1, 2, 3ad, 4,5,6
- Ex 17G #1cde;
- IB Packet #6