T5.2 - Scalar (Dot) Product of Vectors

IB Math SL1 - Santowski

(A) Review

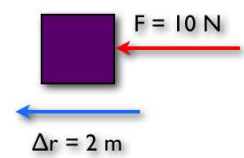
- Operations with Vectors:
- (1) Add/subtract
- (2) multiply by scalar
- (3) HOW do you multiply vectors (if it even means anything in the first place????)

(B) Work (Mini physics lesson)

- The amount of work done by a force on an object (in physics) depends upon:
  - (1) the amount of force applied
  - (2) the distance through which the force is applied (displacement)
  - (3) the direction of the applied force relative to the displacement
- Now, both force and displacement are VECTOR quantities in that both have an associated direction ➔ so what does a vector multiplication mean and how do you do it??

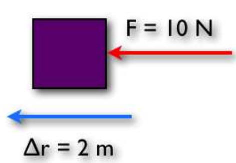
(B) Work (Mini physics lesson)

- Let's keep things simple ➔ if the force applied and the displacement are **IN THE SAME** direction
- I apply a force of 10 Newtons (forward) to move an object 2 meters forward ➔ How much work has the force done in moving the box??
- Diagram



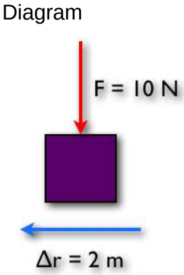
(B) Work (Mini physics lesson)

- Let's keep things simple ➔ if the force applied and the displacement are **IN THE SAME** direction
- I apply a force of 10 Newtons (forward) to move an object 2 meters forward ➔ How much work has the force done in moving the box?? ➔ **20 joules of work**
- Diagram



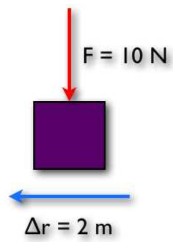
(B) Work (Mini physics lesson)

- Let's make things slightly harder ➔ if the force applied and the displacement are **IN PERPENDICULAR** directions
- I apply a force of 10 Newtons (downward) to move an object 2 meters forward ➔ How much work does the force do in moving the box??
- Diagram



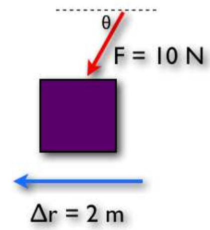
(B) Work (Mini physics lesson)

- Let's make things slightly harder ➔ if the force applied and the displacement are **IN PERPENDICULAR** directions
- I apply a force of 10 Newtons (downward) to move an object 2 meters forward ➔ How much work does the force do in moving the box?? ➔ **0 joules of work**
- Diagram



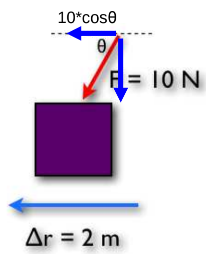
(B) Work (Mini physics lesson)

- Now things get difficult ➔ if there is **an angle (0°< θ<90°)** between the force applied and the displacement
- I apply a force of 10 Newtons (at an angle of θ° to the horizontal) to move an object 2 meters forward ➔ How much work has the force done in moving the box??
- Diagram



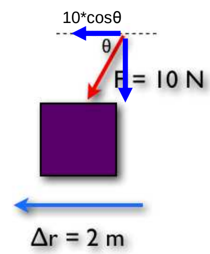
(B) Work (Mini physics lesson)

- I apply a force of 10 Newtons (at an angle of θ to the horiaontal) to move an object 2 meters forward ➔ How much work has the force done in moving the box??
- So we need to work with the **VECTOR COMPONENTS** of our force vector to see **HOW MUCH** of the 10 N of force actually moves the box forward
- It turns out that only the **horizontal component of 10cosθ** moves the box forward
- Diagram



(B) Work (Mini physics lesson)

- I apply a force of 10 Newtons (at an angle of θ to the horiaontal) to move an object 2 meters forward ➔ How much work has the force done upon the box??
- It turns out that only the **horizontal component of 10cosθ** moves the box forward
- So the work done by the force on the box is (10cosθ)*(2) or (10)(2)cosθ
- Diagram



(B) Work (Mini physics lesson) - Summary

- When we deal with vector quantities that are being multiplied (i.e. force and displacement), it is **only the components of the two vectors that are in the same direction that matter** ➔ Hence, the idea of the displacement vector, ***r***, and the horizontal component of the force vector, ***F***cosθ
- So this is how we will understand the multiplication of 2 vectors
- Since WORK is a SCALAR quantity (no direction associated with it), we shall use the term SCALAR PRODUCT to describe this one aspect of vector multiplication

$$F \bullet r = |F||r|\cos\theta \quad \text{or more generally} \quad A \bullet B = |A||B|\cos\theta$$

(C) Scalar (Dot) Product

- So the formula for "multiplying" vectors is

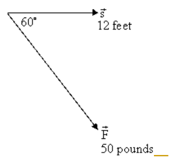
$$A \bullet B = |A||B|\cos\theta$$

$$W = f \bullet d = |f||d|\cos\theta$$

- Where the symbol |***A***| or |***B***| meam the magnitude of vectors ***A*** and ***B***

(C) Scalar (Dot) Product - Examples

- (1) How much work is done by a constant force of 50 Newtons when this force is applied at an angle of 60º to pull a 12 meter sliding metal door shut. The diagram shown below illustrates this situation



(C) Scalar (Dot) Product - Examples

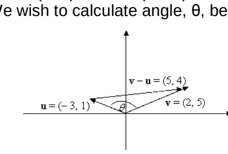
- (2) A force of 20 Newtons is applied to an object at an angle of -45º **with the horizontal**. The object is pulled 10 meters at an angle of 25º **with the horizontal**. How much work is done while moving the object?
- (HINT: Draw a diagram)

(C) Scalar (Dot) Product

- So the formula for "multiplying" vectors is $A \bullet B = |A||B|\cos\theta$
- But what if the vectors are given in component form?
- In general, if **u** = (u_x, u_y) and **v** = (v_x, v_y) and, the dot product is u·v = $u_x v_{x+} u_y v_y$
- The PROOF of this is left up to the STUDENT!!! HAHAHAHAH
- And then in 3 space, if **u** = (u_x, u_y, u_z) and **v** = (v_x, v_y, v_z) and, the dot product is u·v = $u_x v_x + u_y v_y + u_z v_z$

Examples

- (1) Let **v** = (2, 5) and **u** = (–3, 2) be two 2 dimensional vectors. The dot product of **v** and **u** would be ?
- (2) Suppose that **v** = (2, 5) and **u** = (–3, 1) as shown in the diagram shown below. We wish to calculate angle, θ, between **v** and **u**.



- (3) What is the angle between **v** = (3, –7) and **u** = (–1, 9)?

(C) Scalar (Dot) Product of Perpendicular Vectors & Parallel Vectors

- If 2 vectors act perpendicular to each other, the **dot product** (ie **scalar product**) of the 2 vectors has value **zero**.
- For the unit vectors **i** (acting in the *x*-direction) and **j** (acting in the *y*-direction), we have the following dot (ie scalar) products (since they are perpendicular to each other): **i • j = j • i** = 0

Examples

- (4) The dot product of (2,-5,1) and (-4,-3,-2) is:
- (5) Given that a = 3i – j + 2k and b = 2i + j – 2k, find a • b and the angle between the 2 vectors
- (6) http://dev.physicslab.org/PracticeProblems/Worksheets/Phy1Hon/Vectors/dotproduct.aspx
- (7) Find the value of *t* such that the vectors **P** = 3 **i** − 5 **j** and **Q** = 4 **i** + *t* **j** are perpendicular to each other.
- (8) Find the value of *m* such that the vectors ***a*** = 2*m**i*** + *m**j*** + 8***k*** and ***b*** = ***i*** + 3*m**j*** – ***k*** are perpendicular

Examples

- (9) Find a vector perpendicular to $\boldsymbol{a} = 3\boldsymbol{i} - 4\boldsymbol{j}$
- (10) find a vector perpendicular to both $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Properties of the Scalar Product

- (a) Closure ➔ NO closure ➔ the dot product of 2 vectors is NOT another vector
- (b) Commutative ➔ a · b = b · a
- (c) Associative ➔ NO since (a · b) · c is undefined
- (d) distributive ➔ A · (B+C) = A · B + A · C

Properties of the Scalar Product

- (a**u** + b**v**) · **w** = (a**u**) · **w** + (b**v**) · **w**, where a and b are scalars
- **u** · **v** = 0 when **u** and **v** are orthogonal.
- **0** · **0** = 0
- $|\mathbf{v}|^2 = \mathbf{v} \cdot \mathbf{v}$
- a (**u**·**v**) = (a **u**) · **v**

Internet Links

- http://www.youtube.com/watch?v=WNuIhXo39_k
- http://hyperphysics.phy-astr.gsu.edu/Hbase/vsca.html
- http://www.sparknotes.com/physics/vectors/vectormultiplication/section1.html
- http://www.netcomuk.co.uk/~jenolive/vect6.html
- http://videolectures.net/mit801f99_lewin_lec03/
- http://www.intmath.com/Vectors/5_Dot-product-vectors-2-dimensions.php
- http://chortle.ccsu.edu/VectorLessons/vch07/vch07_1.html
- http://cnx.org/content/m14513/latest/

Homework

- HW :
- Ex 15H #1b, 2a,3a, 4ae, 5a, 6a, 8a, 10a, 12c;
- Ex 16G #1be, 4, 7, 9, 11d, 13;
- IB Packet #1, 4, 5, 7