T5.1 – Geometric Vectors

IB Math SL1 - Santowski

(A) Introduction

- We have considered vectors as directed line segments and investigated vectors:
- (a) visually as scale drawings with a particular angle/direction associated with them
- (b) algebraically in terms of x- and y-components as well as i- and j- components
- (c) Now let's place our vectors into the Cartesian plane and combine a visual representation with an algebraic representation as we introduce **ordered pairs** with which to work with vectors

(B) Visual Representation

- Let's consider the two points: A(-3,2) which will be the starting point (or tail) of the vector and the point B(1,-1) which will be the ending point (or head) of the vector.
- So, we have constructed the vector $\vec{v} = \overrightarrow{AB}$
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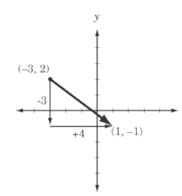
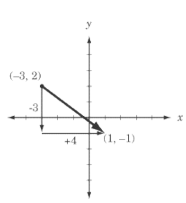


(C) Working with Geometric Vectors

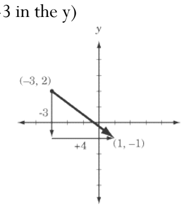
- Since $\vec{v} = \overrightarrow{AB}$ is defined on the Cartesian plane, we can determine:
- (a) its components
- (b) its length
- (c) its direction



(C) Working with Geometric Vectors

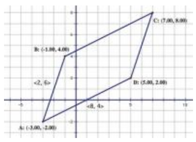
- Since $\vec{v} = \overrightarrow{AB}$ is defined on the Cartesian plane, we can determine:
- (a) its components (+4 in the x and -3 in the y)
- (b) its length $\sqrt{(-4)^2 + (3)^2}$
-
- (c) its direction

$\theta = \tan^{-1}\left(\frac{-3}{4}\right) = -37^\circ \Rightarrow$ bearing 127°



(D) Working with Geometric Vectors – Geometric Shapes

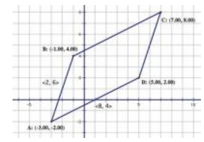
- Given the following diagram, use vector methods prove that the figure is a parallelogram



(D) Working with Geometric Vectors – Geometric Shapes

- Given the following diagram, use vector methods to prove that the figure is a parallelogram

- HINT: What does vector equality mean? How can you show 2 vectors are equal?



(D) Working with Geometric Vectors – Geometric Shapes

- Given the points A(-3,2), B(2,1) and C(-1,-4), find the position of the fourth point such that ABCD is a parallelogram

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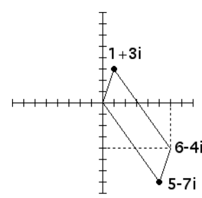
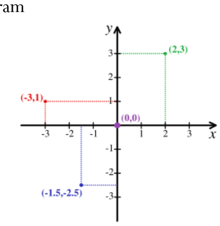


(D) Working with Geometric Vectors – Geometric Shapes

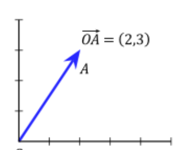
- Given the following diagram, use vector methods determine the position of the 4th point so that the figure is a parallelogram



(E) Position Vectors

- In introducing the idea of a position vector, we now change from **free vectors** (the vectors position in space is NOT considered) to **fixed vectors** (which start at a SPECIFIC point and are thus fixed in space)

- The most convenient fixed point ➔ the origin



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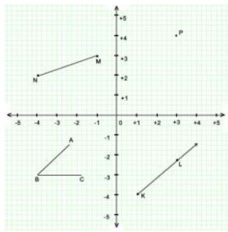
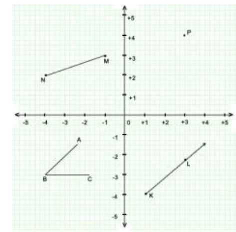


- Consider point P ➔ and construct the position vector $\vec{OP}$

(E) Position Vectors

- Now construct the 2 position vectors $\vec{OM}$ $\vec{ON}$
- So the three vectors can be connected via algebraic operations:

$$\vec{OM} + \vec{MN} = \vec{ON}$$

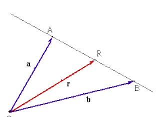


- and our vector can be viewed as a result of a vector subtraction

$$\vec{MN} = \vec{ON} - \vec{OM}$$

(F) Collinear Points

- We can work with position vectors to prove that three points are collinear (in this case A, R, B)



- Let's work with :
- (i) A(2,1), R(4,7), & B(12,16)
- (ii) A(2,1), R(4,7), & B(12,12)

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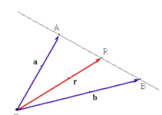
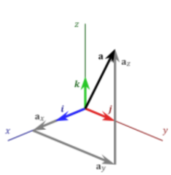
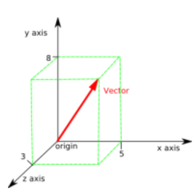


- (ii) A(2,1), R(4,7), & B(12,12)
- Conclusion to be made ➔ 3 points are collinear if one position vector can be written as sum of the other 2 position vectors
- ➔ i.e. $\boldsymbol{r}$ = as + t$\boldsymbol{b}$

(G) 3D Space

- Co-ordinate geometry can also be used to introduce 3D space as we "extend" our Cartesian plane into a third dimension as we consider our vector (a_x, a_y, a_z)





Homework

- HW –
- Ex 15C.1 #1bc, 2ade;
- Ex 15C.2 #1f, 2ef, 3;
- Ex 15C.3 #1df;
- Ex 15C.4 #1ace, 2fgh;
- Ex 15E #1,2, 3, 4, 5ab, 6,7