5.1 – Vector Components

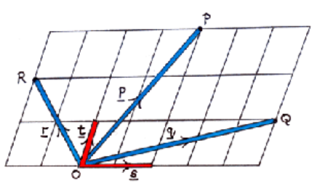
IB Math SL1 - Santowski

(A) Vector Components

- The general point that we will be making now is we want a non-visual way of working with vectors so we don't have to draw them to work with them.
- So first, let's make sure that our vectors are all constructed in the same 2D plane
- Then if we could describe any given vectors in terms of some chosen reference vectors, then we could deal with vectors by using algebra rather than using graphic representations.

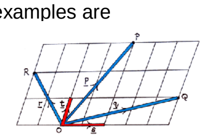
(A) Vector Components

- For example, in the drawing below I have chosen **s** and **t** as my two reference vectors.
- From them we can build up a grid which makes it possible to describe any vector in the plane (like **p**, **r**, **q**) in terms of **s** and **t**.



(A) Vector Components

- You can see that the three examples are described by
- **p** = **s** + 3**t**,
- **q** = (5/2)**s** + **t** and
- **r** = - **s** + 2**t**.



- As an example of working with these vectors, we have:
- **q** + **r** = {(5/2)**s** + **t**} + {- **s** + 2**t**} = (3/2)**s** + 3**t**

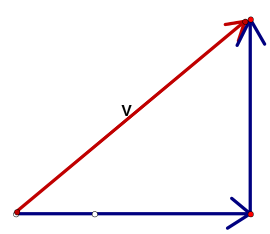
(A) Vector Components - Notations

- So if we change our perspective on vectors from visual to algebraic, let's likewise change our notations:
- So let's use another set of convenient reference vectors ➔ perpendicular vectors parallel to the x- and y-axis
- So now a vector of magnitude 5 units, going in a direction of 37°, can be viewed as **the result of the combination of two perpendicular vectors** ➔ one 4 units along the x-axis and another 3 units along the y-axis
- So the notation to describe this vector will be $\vec{v} = \begin{bmatrix}4\\3\end{bmatrix}$ or $\begin{pmatrix}4\\3\end{pmatrix}$

(A) Vector Components - Notations

- So now a vector of magnitude 5 units, going in a direction of 37°, can be viewed as the result of the combination of two perpendicular vectors ➔ one 4 units along the x-axis and the second 3 units along the y-axis
- So the notation to describe this vector will be

$\vec{v} = \begin{bmatrix}4\\3\end{bmatrix}$ or $\begin{pmatrix}4\\3\end{pmatrix}$



(B) Vector Components - Examples

- So now we have an algebraic form of a vector which we can (i) graphically illustrate, (ii) add, subtract and scalar multiply
- (i) Draw a diagram illustrating the following vectors

$$a=\begin{bmatrix}-4\\6\end{bmatrix}\quad b=\begin{bmatrix}3\\-7\end{bmatrix}\quad c=\begin{bmatrix}-2\\-1\end{bmatrix}$$

(B) Vector Components - Examples

- (ii) Draw each vectors
- (iii) Add, subtract & multiply and draw the resultant vector

$$(a)\ \begin{bmatrix}2\\4\end{bmatrix}+\begin{bmatrix}-5\\3\end{bmatrix}\qquad (b)\ \begin{bmatrix}-3\\12\end{bmatrix}-\begin{bmatrix}-2\\-5\end{bmatrix}$$

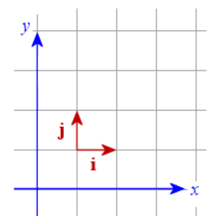
$$(c)\ 3\begin{bmatrix}2\\-1\end{bmatrix}+2\begin{bmatrix}-4\\1\end{bmatrix}\qquad (d)\ -2\begin{bmatrix}5\\-2\end{bmatrix}+5\begin{bmatrix}-1\\-1\end{bmatrix}$$

(C) Vector Components – Unit Vectors

- So if we change our perspective on vectors from visual to algebraic, let's likewise change our notations:
- So let's use other convenient reference vectors ➔ the x- and y-axis
- But let's introduce one last idea ➔ the idea of a unit vector
- We will define 2 unit vectors ➔ one along/parallel to the x-axis (***i***) and one along/parallel to the y-axis (***j***)
- Each unit vector will be 1 unit in length

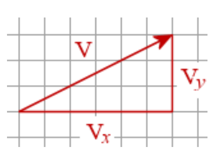
(C) Vector Components – Unit Vectors

- So a visual representation of the unit vectors would be:
- We will define 2 unit vectors ➔ one along/parallel to the x-axis (***i***) and one along/parallel to the y-axis (***j***)
- Each unit vector will be 1 unit in length



(C) Vector Components – Unit Vectors

- In an earlier example, we had the following vector:
- We could write the **components** of the vector **V** as follows.
- **V***x* = 6 **i**
- **V***y* = 3 **j**
- So we can write the vector **V** using **unit vectors** as follows:
- **V** = 6 **i** + 3 **j**



(D) Unit Vectors - Examples

- So now we have an algebraic form of a vector which we can (i) graphically illustrate, (ii) add, subtract and scalar multiply
- (i) Draw a diagram illustrating the following vectors

$$a=-5\vec{i}+4\vec{j}\qquad b=7\vec{i}-4\vec{j}\qquad c=-3\vec{i}-\vec{j}$$

(D) Vector Components - Examples

- (ii) Draw each vectors
- (iii) Add, subtract & multiply and draw the resultant vector

$(a)\ a = (2\vec{i} - 4\vec{j}) + (-5\vec{i} + 2\vec{j})$ $(b)\ a = (-4\vec{i} + 12\vec{j}) - (-2\vec{i} - 5\vec{j})$

$(c)\ a = 3(2\vec{i} - \vec{j}) + 2(-4\vec{i} + 3\vec{j})$ $(d)\ a = -2(5\vec{i} - 3\vec{j}) + 5(-\vec{i} - \vec{j})$

(D) Vector Components - Examples

- (a) Determine the value of k if $\vec{u} = \begin{bmatrix} -\frac{2}{5} \\ k \end{bmatrix}$ is a unit vector
- (b) Illustrate ***u*** with a diagram, given your solution to question (a)
- (c) Does a unit vector HAVE to be parallel to the co-ordinate axes??
- (d) Find a unit vector in the direction of 3***i*** – 4***j***

(E) Homework

- Ex 15C.1 #1bc, 2ade;
- Ex 15C.2 #1f, 2ef, 3;
- Ex 15C.3 #1df;
- Ex 15C.4 #1ace, 2fgh;
- Ex 15G #1abc, 2ad, 3bc, 4bd, 6a;