T5.1 – Basics of Vectors – Geometry of Vectors

IB Math SL - Santowski

(A) What are Vectors?

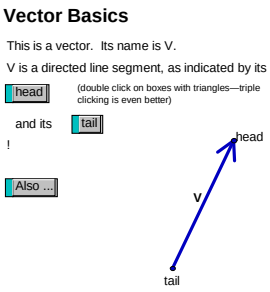
- Some physical properties, such as temperature or area, are described completely by **their magnitude** and so only need a number (called a **scalar**) to represent them.
- But there are other physical quantities, such as force, velocity or acceleration, for which we must know **direction as well as size or magnitude** in order to work with them.
- It is often very helpful to **represent** such quantities by directed lines called **vectors** ➔ so vectors carry the physical information of both magnitude and direction
- **Vectors** have direction, and obey different rules of arithmetic.

(B) Representing Vectors – Notations

- **Notation**
- • Scalars : ordinary or italic font (*m*, *q*, *t* . . .)
- • Vectors :
- (1) Boldface font (**v**, **a**, **F** . . .)
- (2) arrow notation ($\vec{v}, \vec{a}, \vec{F}$)
- (3) underline (v, a, F . . .)

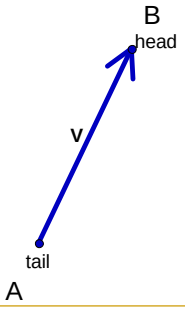
(B) Representing Vectors - Graphically

- Graphically, vectors can be represented as directed line segments (rays) with a tail (starting point) and a head (end point)



(B) Representing Vectors – Notations from Graphic Representations

- Graphically, vectors can be represented as directed line segments (rays)
- Since a vector has a head and a tail, we can label the points at which its head and tail are located and use this to name the vector ➔ $\vec{v} = \overrightarrow{AB}$



(B) Representing Vectors - Graphically

- Since vectors are "directed line segments" with magnitude & direction, they can be drawn or represented with SCALE diagrams
- Ex. A force of 20 N south
- Ex. A displacement of 30 km E30°S (or a bearing of 120°)

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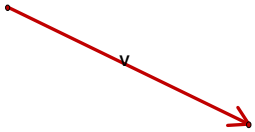
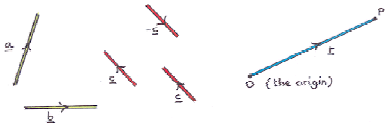


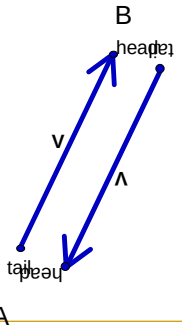
(C) Vector Concepts – Vector Equality

- (a) Vector Equality ➔Two vectors are equal if and only if they are equal in both magnitude and direction.
- Equal vectors are also PARALLEL (and equal in length obviously)
- 

(C) Vector Concepts – Negative Vectors

- (b) Negative Vectors ➔ If c is a vector, then the vector -c is defined as having the same magnitude but the reverse direction to c.
- Now Point A is where the original vector starts (and ends at Point B)➔ the negative vector starts at point B and ends at A ➔

$$\overrightarrow{AB} = -\overrightarrow{BA}$$



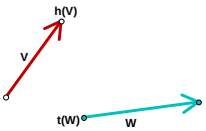
(D) Vector Addition – Geometric Perspective

- We can visualize the addition of vectors as follows:

Definition of Vector Addition

To add vectors V and W, take the TAIL TW of W and move it to the HEAD HV of V by parallel displacement.

Do it!



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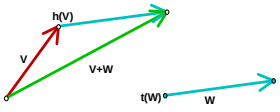
Then V + W is the vector whose tail is at the tail of V and head is at the head of W.

Show V+W



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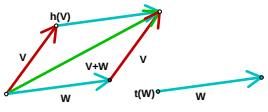
If we fill in another copy of V and W then we have a parallelogram.. Its diagonal is V+W.

Show parallelogram

This is called the PARALLELOGRAM law of addition



(D) Vector Addition – Geometric Perspective

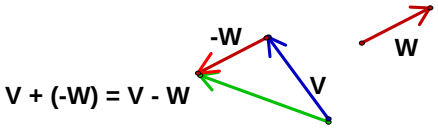
- Link #1
- http://www.slu.edu./classes/maymk/SketchpadApplets/AddVectors.html1
- End D,H block

(E) Vector Subtraction – Geometric Perspective

- We have 2 ways that we can view/understand vector subtraction:
- (a) Adding a negative vector: $\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$

Given V and W, determine -W and then V + (-W) = V - W.

try it

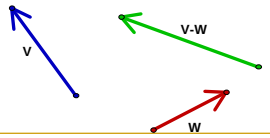


(E) Vector Subtraction – Geometric Perspective

- We have 2 ways that we can view/understand vector subtraction:
- (b) Consider V-W as a vector that gets added to W to give V: $\vec{w} + (\vec{v} - \vec{w}) = \vec{v}$

Given V and W, V-W is that vector which, when added to W yields V:
W + (V-W) = V.

try it



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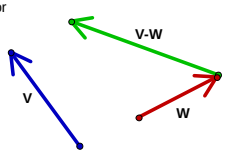

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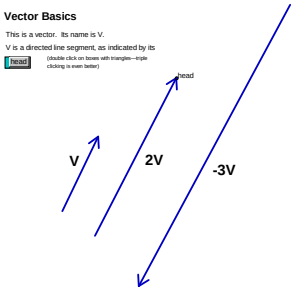
(F) Vector Operations – Scalar Multiplication

- Scalar multiplication can be seen as changing the length (or magnitude) and/or direction of a vector



(F) Examples – Operations with Vectors

- Link #1
- http://teachersites.schoolworld.com/webpages/NiedzielaM/files/graphical%20vector%20addition2.pdf
- Link #2
- http://www.teachnet-uk.org.uk/2006%20Projects/Maths-KS4-5_catch-up/straight_lines-vectors/Worksheet-vectors.doc1

Internet Links for Extra Help

- http://www.netcomuk.co.uk/~jenolive/homevec.html
- http://mathforum.org/~klotz/Vectors/index.html#vectors1 (Require Geometers SketchPad program to see interactive discussions)
- http://physwww.mcmaster.ca/~jopkoam/md3.pdf

Homework

- HW –
- Ex 15A.1 #1ad;
- Ex 15A.2 All;
- Ex 15B.1 #1acef, #2ac;
- Ex 15B.2 #1ac, 2b, 4be, 5acef, 6;
- Ex 15B.3 #1abef, 2de
- Note: All Ex 15B #1, 2 on graph paper
- Links to Handouts: http://mrsantowski.tripod.com/2009MathSLY1/Assignments/graphical_vector_addition2.pdf
- http://mrsantowski.tripod.com/2009MathSLY1/Assignments/wsvectors.pdf