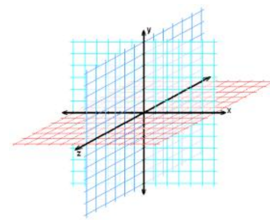
T5.1 – 3D Vectors

IB Math SL1 - Santowski

(A) 3D Grid

- To be able to work in 3 space, we will set up a "grid" so that we can discuss points & vectors
- We have our origin as well as 3 mutually perpendicular axes called the x-axis, y-axis and the z-axis
- We will then also have 3 co-ordinate planes ➔ the xy plane, the xz plane and the yz plane
- Here is an illustration of our "grid"



(A) 3 Space – Co-ordinate Planes

- The ***x-y* plane** is horizontal in our diagram above and shaded green. It can also be described using the equation *z* = 0, since all points on that plane will have 0 for their *z*-value.
- The ***x-z* plane** is vertical and shaded pink above. This plane can be described using the equation *y* = 0.
- The ***y-z* plane** is also vertical and shaded blue. The *y-z* plane can be described using the equation *x* = 0.
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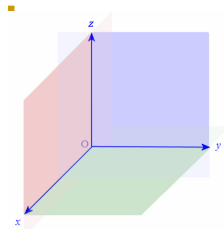

(B) Points and Vectors in 3 Space

- Any point in 3 space can be described by an ordered triplet ➔ (x,y,z) which correspond to the point's location along the x-, y-, and z-axis (as in 2 space)
- Once we have a point located, say P, we can construct the corresponding position vector, OP
- And since we have a vector now drawn, we can determine its length, and write it in component form
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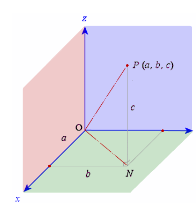
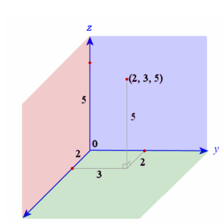


(B) Points and Vectors in 3 Space

- So consider the point A(2,3,5), we can determine its length:
- We can write it in component form as:
- Or we can find its midpoint



(B) Points and Vectors in 3 Space

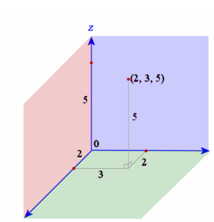
- So consider the point A(2,3,5), we can determine its length:

$$|\vec{OP}| = \sqrt{(2-0)^2+(0-3)^2+(0-5)^2} = \sqrt{38}$$

- We can write it in component form as: $\vec{OP} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$
- Or we can find its midpoint

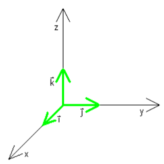
$$MP = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

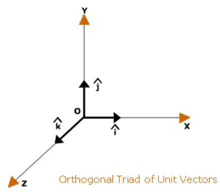
$$MP = \left(1, \frac{3}{2}, \frac{5}{2}\right)$$



(C) Unit Vectors in 3 Space

- We can introduce our special unit vectors in 3 space as well the unit vector parallel to the z axis ➔ the ***k*** vector

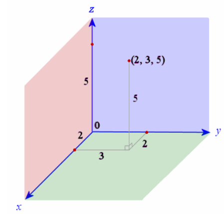




(C) Unit Vectors in 3 Space

- So revisit the point A(2,3,5),
- We can write it in component form as: $\vec{OP} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$
- Or we can write it in unit vector notation as

$$\vec{OP} = 2\vec{i} + 3\vec{j} + 5\vec{k}$$



(D) 3D Vectors & Co-ordinate Geometry

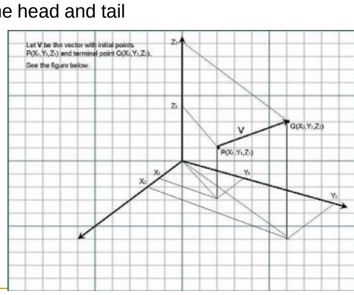
- As we did in 2 space, we can work with specific points for the head and tail
- So if P(2,3,5) and Q(-1,6,-2), determine $\vec{PQ}$ in component form and in unit vector form.
- Determine the length of $\vec{PQ}$

(D) 3D Vectors & Co-ordinate Geometry

- As we did in 2 space, we can work with specific points for the head and tail
- So if P(x_1,y_1,z_1) and Q(x_2,y_2,z_2), determine $\vec{PQ}$ in component form and in unit vector form.
- Determine the length of $\vec{PQ}$

(D) 3D Vectors & Co-ordinate Geometry

- As we did in 2 space, we can work with specific points for the head and tail



(E) Algebra with 3D Vectors

- Working with 3D vectors algebraically is very similar to working with 2D vectors
- Let $a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$ and $b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$
- (1) addition/subtraction (2) scalar multiplication

$$a + b = \begin{bmatrix} a_x + b_x \\ a_y + b_y \\ a_z + b_z \end{bmatrix} \quad a - b = \begin{bmatrix} a_x - b_x \\ a_y - b_y \\ a_z - b_z \end{bmatrix} \quad ka = k\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} ka_x \\ ka_y \\ ka_z \end{bmatrix}$$

(E) Algebra with 3D Vectors

- To help make our algebra work with vectors easier, let's make certain generalizations (rules) that are true for vector algebra

- Commutative property ➔ ***a*** + ***b*** = ***b*** + ***a***
- Associative property ➔ (***a*** + ***b***) + ***c*** = ***a*** + (***b*** + ***c***)
- Additive identity property ➔ ***a*** + 0 = 0 + ***a*** = ***a***
- Additive inverse property ➔ ***a*** + (-***a***) = (-***a***) + ***a*** = 0

(F) Examples

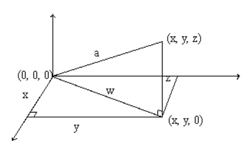
- Cirrito/HH text

(F) Examples

- Given the point P(4,5,-7), determine:
- (a) its position vector
- (b) a vector that is equal to OP
- (c) a vector that is anti-parallel to OP
- (d) how far the point P is from:
  - (i) the origin
  - (ii) the XZ plane
  - (iii) the x-axis

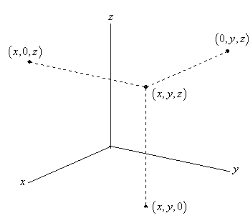
(F) Examples

- The following diagram may help in Q(d)



(F) Examples

- The following diagram may help in Q(d)



(E) Examples

- Let's go back to parallelograms:

- Given A(-1,2,5) and B(2,0,3) and C(-3,1,0) and D(x,y,z)

- Find the co-ordinates of D in parallelogram ABCD if:
  - (i) $\overrightarrow{AB} = \overrightarrow{DC}$ (ii) $\overrightarrow{AC} = \overrightarrow{BD}$

Homework

- HW:
- Ex 16A #1c, 2c, 4a;
- Ex 16B.1 #2,3;
- Ex 16B.2 #2a, 3a, 5a
- Ex 16C #1h,2c, 5-8