T4.4 – Linear Systems & Matrices

IB Math SL - Santowski

Fast Five

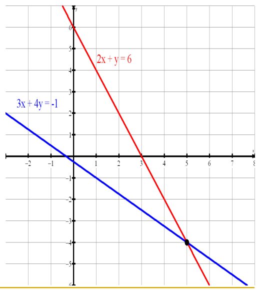
- (1) If ***A*** and ***B*** are both *n* by *n* matrices, is ***AB*** = ***BA***?
- (2) Solve the equation 8x = 16 without using division
- (3) Multiply the matrices $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$

(A) Linear Systems

- At this point in math, you have 2 major ways to solve linear systems:
- (a) Algebraic (Elimination or Substitution)
- (b) Graphic (look for intersection point)
- So solve the system defined by:
- L_1: 2x + y = 6
- L_2: 3x+4y = -1

(A) Linear Systems

- L_1: 2x + y = 6
- L_2: 3x+4y = -1
- From L1 ➔ y = 6-2x
- Sub into L2:
- 3x + 4(6 – 2x) = -1
- -5x = -25
- so x = 5
- and y = -4



(A) Linear Systems & Matrices

- So let's combine linear systems & matrices
- So HOW do you write the system below as a matrix??? (see fast five)
- L_1: 2x + y = 6
- L_2: 3x+4y = -1

(A) Linear Systems & Matrices

- So the system
- L_1: 2x + y = 6 and L_2: 3x+4y = -1
- Now becomes a matrix multiplication as ….
- $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$

where:

$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \Rightarrow$ coefficient matrix

$x = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow$ variable matrix

$B = \begin{bmatrix} 6 \\ -1 \end{bmatrix} \Rightarrow$ constant matrix

(A) Linear Systems & Matrices

- So the system
- L_1: 2x + y = 6 and L_2: 3x+4y = -1
- Now becomes a matrix multiplication as ….
- $$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

  so we have the matrix multiplication :

  $Ax = B$

(A) Linear Systems & Matrices

- So how do we multiply matrices for the viewpoint of isolating an unknown (or isolating the variables matrix?)
- Ax = B

(A) Linear Systems & Matrices

- So how do we multiply matrices for the viewpoint of isolating an unknown (or isolating the variables matrix?) ➔ USE THE INVERSE!!!
- But HOW????? ➔ recall that the order in which you multiply matrices IS IMPORTANT

(B) Matrix Multiplication (A Review)

- Define the following matrices on your TI-83/4

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \quad x = C = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

- so the system you are investigating is:

$$\left.\begin{matrix} L_1 : 3x - y = 5 \\ L_2 : -x + 4y = 2 \end{matrix}\right\} \quad (2,1)$$

- So carry out the following multiplications ➔ Which statements are true?

$$\begin{aligned} &\text{(a) } Ax = B \\ &\text{(b) } (Ax)A^{-1} = BA^{-1} \\ &\text{(c) } (AA^{-1})x = BA^{-1} \\ &\text{(d) } (A^{-1}A)x = A^{-1}B \end{aligned}$$

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(B) Matrix Multiplication (A Review)

- So, since statements (a) and (d) are true, how does this relate to solving equations using matrices????

$$\begin{aligned} &\text{(a) } Ax = B \\ &\text{(d) } (A^{-1}A)x = A^{-1}B \Rightarrow (I)x = A^{-1}B \Rightarrow x = A^{-1}B \end{aligned}$$

- Recall that the product of a matrix and its inverse is ***I***
- So what we have effectively accomplished is ISOLATING the variable matrix WITHOUT using division!!

(C) Matrices and Linear Equations

- So let's consolidate everything:
- So use matrices to solve the system defined by:
- L_1: 2x + y = 6
- L_2: 3x+4y = -1

(C) Matrices and Linear Equations

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$$

$$Ax = B \qquad \Rightarrow A^{-1} = \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix}$$

$$\left(A^{-1}A\right)x = A^{-1}B$$

$$\left(\begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\right) \times \begin{bmatrix} x \\ y \end{bmatrix} = \left(\begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 6 \\ -1 \end{bmatrix}\right)$$

$$Ix = x = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 6 \\ -1 \end{bmatrix}\right)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

(D) Examples – Matrices & Linear Systems

- (1) Solve the system: –x+5y = 4 and 2x + 5y = -2. Interpret your solution.
- (2) Solve the system: -x + 2y = 4 and 5x – 10 = 6. Interpret your solution.
- (3) Solve the system –x + 2y = 2 and 4x – 8y = -8. Interpret your solution.
- (4) Solve the system x + 2y – z = 6 and 3x + 5y – z = 2 and -2x – y – 2z = 4. Interpret your solution.

(D) Examples – Matrices & Linear Systems

- Consider the system 2x + *k*y = 8 and 4x – y = 11.
- (a) Calculate det[A]
- (b) for what value(s) of *k* does the system have a unique solution? Find the unique solution.
- (c) for what value(s) of *k* does the system not have a unique value. How many solutions does it have in this case?

(D) Examples – Matrices & Linear Systems

- (a) Show that if ***AX*** = ***B***, then ***X*** = $\boldsymbol{A}^{-1}\boldsymbol{B}$ whereas if ***XA*** = ***B***, then ***X*** = $\boldsymbol{BA}^{-1}$.
- (b) Solve for X if $X\begin{bmatrix} 1 & 2 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 14 & -5 \\ 22 & 0 \end{bmatrix}$
-
- (c) Solve for X if $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} X = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

(D) Examples – Matrices & Linear Systems

- Given that

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$$

- Solve for ***X*** if ***AXB*** = ***C***.
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(D) Examples – Matrices & Linear Systems

We want 10 L of gasoline containing 2% additive. We have drums of the following:

*gasoline without additive

*gasoline with 5% additive

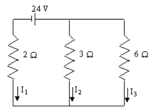
*gasoline with 6% additive

We need to use 4 times as much pure gasoline as 5% additive gasoline. How much of each is needed?

Always check your solutions!

(D) Examples – Matrices & Linear Systems

Find the electric currents shown by solving the matrix equation (obtained using Kirchhoff's Law) arising from this circuit:



$$\begin{pmatrix} I_1 + I_2 + I_3 \\ -2I_1 + 3I_2 \\ -3I_2 + 6I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 24 \\ 0 \end{pmatrix}$$

Homework

- Matrices & Systems:
- Ex 14H #2ad, 8acf;
- Ex 14I #1a, 3ab,4b, 7;
- Ex 14K #2a; Ex 14L #5a, 8;

- 3x3 matrices
- Ex 14H #5;
- Ex 14I #5ac, 6, 8b;
- Ex 14J#1agh, 3, 6a

- IB Packet #1,2,4,5,7