T4.1 – Introduction to Matrices

IB Math SL - Santowski

(A) Matrices – Data Storage

- I need info about your ➔ height, weight, age
- Now, I need to ORGANIZE and PRESENT my data:
- Ex ➔ ordered triple: (185 cm, 90kg, 29 years)
- Ex ➔ row vector form ➔ $\begin{bmatrix} 185 & 90 & 29 \end{bmatrix}$
- Ex ➔ column vector form ➔ $\begin{bmatrix} 185 \\ 90 \\ 29 \end{bmatrix}$
- Each entry into our ordered triples/vectors is called an **element**
- **Q** ➔ does the order within the triple/vector matter??

(A) Matrices – Data Storage

- Now let's add to our data:
- Ex ➔ ordered triple: (170 cm, 70kg, 29 years)
- Ex ➔ ordered triple: (145 cm, 35kg, 11 years)
- Ex ➔ ordered triple: (140 cm, 30kg, 9 years)
- Ex ➔ ordered triple: (130 cm, 25kg, 8 years)

$$\begin{bmatrix} 185 & 90 & 29 \\ 170 & 70 & 29 \\ 145 & 35 & 11 \\ 140 & 30 & 9 \\ 130 & 25 & 8 \end{bmatrix}$$

- And then let's combine our "vectors" into a new format: ➔ Basically, we have all of our information organized into one arrangement called a matrix.

$$\begin{bmatrix} 185 & 170 & 145 & 140 & 130 \\ 90 & 70 & 35 & 30 & 25 \\ 29 & 29 & 11 & 9 & 8 \end{bmatrix}$$

(B) Matrices – Key Terms

- A **real matrix** is an arrangement of real numbers into rows and columns.
- The real numbers are called the **elements** of the matrix.
- Each **row** means something in context
- Each **column** means something in context

- Our example:

	h	wt	age
MrS	185	90	29
MrsM	170	70	29
Alex	145	35	11
Andrew	140	30	9
Ian	130	25	8

(B) Matrices – Key Terms

- This matrix is referred to as a **5 by 3 matrix** (often written 5x3) because there are 5 rows and 3 columns.
- Therefore, the **dimensions** of this matrix are 5 by 3.
- The dimensions of a matrix tell you the "size" of the matrix because they tell you the number of rows and columns in the matrix.
- By convention, we list the number of rows before the number of columns.
- **Definition** ➔The **dimensions** of a matrix are the number of rows and columns (listed in that order) of the matrix.

- Our example:

	h	wt	age
MrS	185	90	29
MrsM	170	70	29
Alex	145	35	11
Andrew	140	30	9
Ian	130	25	8

(B) Matrices – Key Terms

- Each element of the matrix is named according to its position.
- Typically, capital letters represent matrices and small letters with subscripts represent elements in the matrix.
- If we name the above matrix *A*, the element 90 is in the position a_{12} (read *a* one two) because it is in row 1 and column 2.
- Also by convention, we list the row number of the element before the column number.
- An element in row *i* and column *j* would be denoted by a_{ij}.
- This gives us a compact way to refer to specific elements of a matrix.

- Our example:

$$A = \begin{bmatrix} 185 & 90 & 29 \\ 170 & 70 & 29 \\ 145 & 35 & 11 \\ 140 & 30 & 9 \\ 130 & 25 & 8 \end{bmatrix}$$

(C) Other Key Terms

- (a) Matrix *B* is the **transpose** of *A*, and *A* is the **transpose** of *B*. **Transposing** a matrix results in writing the columns as rows and the rows as columns, but what really happens is that element a_{ij} is placed in the position b_{ji} of the new matrix.

$$A = \begin{bmatrix} 185 & 90 & 29 \\ 170 & 70 & 29 \\ 145 & 35 & 11 \\ 140 & 30 & 9 \\ 130 & 25 & 8 \end{bmatrix}$$

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$$A = \begin{bmatrix} 185 & 90 & 29 \\ 170 & 70 & 29 \\ 145 & 35 & 11 \\ 140 & 30 & 9 \\ 130 & 25 & 8 \end{bmatrix} \qquad B = A^T = \begin{bmatrix} 185 & 170 & 145 & 140 & 130 \\ 90 & 70 & 35 & 30 & 25 \\ 29 & 29 & 11 & 9 & 8 \end{bmatrix}$$

(C) Other Key Terms

$$S = \begin{bmatrix} 9 & 2 & 5 & 1 \\ 2 & 7 & 0 & 8 \\ 5 & 0 & 4 & 6 \\ 1 & 8 & 6 & 3 \end{bmatrix}$$

- (b) a SQUARE matrix ➔ matrix *S*, given above, has another special property; it is a **square matrix** because *S* has the same number of rows as columns.
- Notice that *S* is a 4 by 4 square matrix.
- We said that the main diagonal for *S* runs from 9 to 3.
- For any square matrix, the **main diagonal** runs from the upper left corner to the lower right corner.

(C) Other Key Terms

$$S = \begin{bmatrix} 9 & 2 & 5 & 1 \\ 2 & 7 & 0 & 8 \\ 5 & 0 & 4 & 6 \\ 1 & 8 & 6 & 3 \end{bmatrix}$$

- (c) A matrix is said to be **symmetric** if $A = A^T$.
- What DOES that MEAN ?????

(C) Other Key Terms

- The addition property of zero for real numbers tells us that $r + 0 = 0 + r = r$.
- There is also an addition property of zero for matrices which states that $A + 0 = 0 + A = A$ where 0 represents the zero matrix of the same dimensions as *A*.
- **Definition ➔** A **zero matrix** is a matrix which has the number 0 for each of its elements.

(D) Matrix Algebra

- If matrices are "data storage" devices,
  - Can we do "algebra" with matrices?
  - What would be the meaning of our matrix algebra?
- Let's consider my family data matrices as individual matrices

$$D = \begin{bmatrix} 185 \\ 90 \\ 29 \end{bmatrix} \quad M = \begin{bmatrix} 170 \\ 70 \\ 29 \end{bmatrix} \quad A = \begin{bmatrix} 145 \\ 35 \\ 11 \end{bmatrix} \quad AJ = \begin{bmatrix} 140 \\ 30 \\ 9 \end{bmatrix} \quad I = \begin{bmatrix} 130 \\ 25 \\ 8 \end{bmatrix}$$

(D) Matrix Algebra

- What does matrix **equality** mean??
- Can you **add** matrices? Under what conditions? (What would D + M or A + AJ + I mean in this context?)
- Can you **subtract** matrices? Under what conditions? (What would D - M or A - AJ or A – I mean in this context?)

$$D=\begin{bmatrix}185\\90\\29\end{bmatrix}\quad M=\begin{bmatrix}170\\70\\29\end{bmatrix}\quad A=\begin{bmatrix}145\\35\\11\end{bmatrix}\quad AJ=\begin{bmatrix}140\\30\\9\end{bmatrix}\quad I=\begin{bmatrix}130\\25\\8\\2\end{bmatrix}$$

(D) Matrix Algebra

- What does matrix **equality** mean ➔ We say that two *m* by *n* matrices, *A* and *B* are **equal** if their corresponding elements are equal. In other words, *A* = *B* if *A* and *B* have the same dimensions and $a_{11} = b_{11}$, $a_{12} = b_{12}$, etc
- So a matrix, B, that would be equal to D would be ??????
- So a matrix, C, that would be equal to A would be ??????

$$D=\begin{bmatrix}185\\90\\29\end{bmatrix}\quad M=\begin{bmatrix}170\\70\\29\end{bmatrix}\quad A=\begin{bmatrix}145\\35\\11\end{bmatrix}\quad AJ=\begin{bmatrix}140\\30\\9\end{bmatrix}\quad I=\begin{bmatrix}130\\25\\8\end{bmatrix}$$

(D) Matrix Algebra

- Can you **add** matrices? Under what conditions? (What would D + M or A + AJ + I mean in this context?)

$$D+M=\begin{bmatrix}185\\90\\29\end{bmatrix}+\begin{bmatrix}170\\70\\29\end{bmatrix}=\begin{bmatrix}355\\160\\58\end{bmatrix}$$

- Can you **subtract** matrices? Under what conditions? (What would D - I or A - AJ mean in this context?)

$$D-I=\begin{bmatrix}185\\90\\29\end{bmatrix}-\begin{bmatrix}139\\25\\8\\2\end{bmatrix}\qquad A-AJ=\begin{bmatrix}145\\35\\11\end{bmatrix}-\begin{bmatrix}140\\30\\9\end{bmatrix}=\begin{bmatrix}5\\5\\2\end{bmatrix}$$

(D) Matrix Algebra

- Can we **multiply** a matrix by a constant? Under what conditions?
- So let's change our context for the matrices
- We have three recipes for breakfast foods. Each recipe feeds three people. The ingredients are as follows:
- Pancakes: 2 cups baking mix, 2 eggs, and 1 cup milk.
- Biscuits: 2 ¼ cups baking mix and ¾ cups milk.
- Waffles: 2 cups baking mix, 1 egg, 1⅓ cups milk, and 2 tablespoons vegetable oil.
- Let's write this in the form of a labeled matrix so that it is easier to read.

(D) Matrix Algebra

- Pancakes: 2 cups baking mix, 2 eggs, and 1 cup milk.
- Biscuits: 2 ¼ cups baking mix and ¾ cups milk..
- Waffles: 2 cups baking mix, 1 egg, 1⅓ cups milk, and 2 tablespoons vegetable oil.
- Let's write this in the form of a labeled matrix so that it is easier to read.

$$R=\begin{bmatrix} & Bm & E & M & O\\ P & 2 & 2 & 1 & 0\\ B & 2\frac{1}{4} & 0 & \frac{3}{4} & 0\\ W & 2 & 1 & 1\frac{1}{3} & 2\end{bmatrix}$$

(D) Matrix Algebra

- We have three recipes for breakfast foods. Each recipe feeds three people. The ingredients are as follows:
- Pancakes: 2 cups baking mix, 2 eggs, and 1 cup milk.
- Biscuits: 2 ¼ cups baking mix and ¾ cups milk.
- Waffles: 2 cups baking mix, 1 egg, 1⅓ cups milk, and 2 tablespoons vegetable oil.
- Now if I want to feed my extended family of 9 people ➔ what must I do with me recipe? ➔ make 3 batches

(D) Matrix Algebra

- Pancakes: 2 cups baking mix, 2 eggs, and 1 cup milk.
- Biscuits: Biscuits: 2 ¼ cups baking mix and ¾ cups milk.
- Waffles: 2 cups baking mix, 1 egg, 1⅓ cups milk, and 2 tablespoons vegetable oil.
-

  Now if I want to feed my extended family of 9 people ➔ what must I do with me recipe? ➔ make 3 batches

$$R=\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix} \quad R=\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix} \quad R=\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix}$$

(D) Matrix Algebra

- So here is our scalar multiplication ….

$$R+R+R=\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix}+\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix}+\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix}=\begin{bmatrix} 6 & 6 & 1 & 0 \\ 6\frac{3}{4} & 0 & 2\frac{3}{4} & 0 \\ 6 & 3 & 4 & 6 \end{bmatrix}$$

$$3R=3\times\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 2 & 1 & 1\frac{1}{3} & 2 \end{bmatrix}=\begin{bmatrix} 6 & 6 & 1 & 0 \\ 6\frac{3}{4} & 0 & 2\frac{3}{4} & 0 \\ 6 & 3 & 4 & 6 \end{bmatrix}$$

(D) Matrix Algebra

- When we multiply a matrix by a real number, we call the real number a **Scalar** and call the operation **scalar multiplication**.
- Scalar multiplication consists of multiplying each element of a matrix by a given scalar.
- If c is a real number and A is a matrix whose (i,j)th element is a_{ij}, then the **scalar product** cA is the matrix whose (i,j)th element is ca_{ij}.

(E) Internet Links

- http://ceee.rice.edu/Books/LA/intro/index.html
- http://www.mrsantowski.com/IBSLY2/Notes/Matrix40.htm
- http://www.mrsantowski.com/IBSLY2/Notes/Matrix421.htm

(F) Homework

- HH Text:
- Ex 14A, Q1-4
- Ex 14B, Q1-8
- Ex 14C, Q1-6