

# T35 - Trigonometric Equations

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## FAST FIVE

- EXPLAIN the difference between the following 2 equations:
- (a)  $\sin(x) = 0.75$
- (b)  $\sin(0.75) = x$
- Now, use you calculator to solve for  $x$  in both equations
- Define “principle angle” and “related acute angle”

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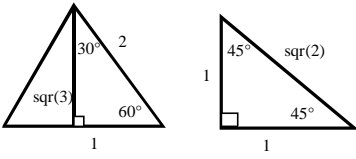
## (A) Review

- We have two key triangles to work with in terms of determining our related acute angles and we can place a related acute angle into any quadrant and then use the CAST “rule” to determine the sign on the trigonometric ratio
- The key first quadrant angles we know how to work with are  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$

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## (A) Review

- The two triangles and the CAST “rule” are as follows:



S	A
T	C

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## (A) Review

- We can set up a table to review the key first quadrant ratios:

$\theta$	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$0$	$0$	$1$	$0$
$30^\circ$ or $\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$45^\circ$ or $\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	$1$
$60^\circ$ or $\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$90^\circ$ or $\pi/2$	$1$	$0$	Undef.

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## (B) Solving Linear Trigonometric Equations

- We will outline a process by which we come up with the solution to a trigonometric equation → it is important you understand WHY we carry out these steps, rather than simply memorizing them and simply repeating them on a test of quiz

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### (B) Solving Linear Trigonometric Equations

- Work with the example of  $\sin(\theta) = -\sqrt{3}/2$
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant the angle must lie
- Step 3: draw a diagram showing the related acute in the appropriate quadrants
- Step 4: from the diagram, determine the principle angles

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### (B) Solving Linear Trigonometric Equations - Solns

- Work with the example of  $\sin(\theta) = -\sqrt{3}/2$
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles (in this case, simply work with the ratio of  $\sqrt{3}/2$ )  $\rightarrow \theta = 60^\circ$  or  $\pi/3$
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant the angle must lie  $\rightarrow$  quad. III or IV in this example

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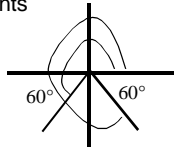
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### (B) Solving Linear Trigonometric Equations

- Step 3: draw a diagram showing the related acute in the appropriate quadrants



- Step 4: from the diagram determine the principle angles  $\rightarrow 240^\circ$  and  $300^\circ$  or  $4\pi/3$  and  $5\pi/3$  rad.

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### (B) Solving Linear Trigonometric Equations

- One important point to realize  $\rightarrow$  I can present the same original equation ( $\sin(\theta) = -\sqrt{3}/2$ ) in a variety of ways:
  - (i)  $2\sin(\theta) = -\sqrt{3}$
  - (ii)  $2\sin(\theta) + \sqrt{3} = 0$
  - (iii)  $\theta = \sin^{-1}(-\sqrt{3}/2)$

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### (C) Further Examples

- Solve the following without a calculator

$$2\cos(\theta) + 2 = 3 \quad \text{for } \theta \in (0, 4\pi)$$

$$\sin(2\theta) - 0.5 = 0 \quad \text{for } \theta \in (0, 2\pi)$$

$$\cos\left(\theta - \frac{\pi}{4}\right) + 1 = 2 \quad \text{for } \theta \in (-2\pi, 2\pi)$$

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### (C) Further Practice

- Solve the following for  $\theta$ :

$$\sin \theta = 0 \quad \text{for } 0 \leq \theta \leq 4\pi$$

$$\sin \theta = 1 \quad \text{for } -2\pi \leq \theta \leq 2\pi$$

$$1 + \cos \theta = 0 \quad \text{for } -\pi \leq \theta \leq 3\pi$$

$$\tan \theta = 0 \quad \text{for } 0 \leq \theta \leq 3\pi$$

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## (C) Further Practice

- Solve without a calculator

$$\begin{aligned}\sqrt{3} + 3\sin x &= 5\sin x \quad \text{for } x \in (0, 4\pi) \\ 8\cos x + 1 &= 2\cos x + 4 \quad \text{for } x \in (-4\pi, 0) \\ \sin x - 4 &= -2\sin x \quad \text{for } x \in (-2\pi, 2\pi) \\ \sin^2 x - 3 &= -3\sin^2 x \quad \text{for } x \in (0, 2\pi)\end{aligned}$$

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## Review – Graphic Solutions

- We know what the graphs of the trigonometric functions look like
- We know that when we algebraically solve an equation in the form of  $f(x) = 0$ , then we are trying to find the roots/zeros/x-intercepts
- So we should be able to solve trig equations by graphing them and finding the x-intercepts/intersection points

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## (D) Modeling Periodic Phenomenon & Trig Equations

The height,  $h$ , of a basket on a water wheel at time  $t$  is given by  $h(t) = \sin(6t)^\circ$ , where  $t$  is in seconds and  $h$  is in metres.

- How high is the basket at 14 s?
- When will the basket first be 0.5 m under water?

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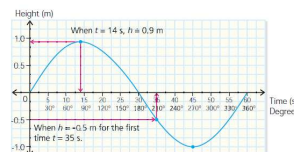
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## (D) Modeling Periodic Phenomenon & Trig Equations

- The values of  $h$  and  $t$  can be interpolated from a graph. Prepare a table. In this case, 5-s intervals were used, although the interval size could be different.

$t$ (seconds)	0	5	10	15	20	25	30	35	40	45	50	55	60
$(6t)^\circ$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$h(t) = \sin(6t)$ (metres)	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Interpolating from the graph gives a value of about 0.9 m, as shown.



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## (D) Modeling Periodic Phenomenon & Trig Equations

The height,  $h$ , of a basket on a water wheel at time  $t$  is given by  $h(t) = \sin(6t)^\circ$ , where  $t$  is in seconds and  $h$  is in metres.

- How high is the basket at 14 s?
- When will the basket first be 0.5 m under water?

The question could also be answered by substituting into the equation.

$$\begin{aligned}h(14) &= \sin(6 \times 14)^\circ \\ &= \sin 84^\circ \\ &\approx 0.995\end{aligned}$$

Substitution gives a more accurate answer in this case. At 14 s, the height is almost 1 m.

A height of 0.5 m under water corresponds to a height of  $-0.5$  m in the model. Therefore,

$$\begin{aligned}h(t) &= \sin(6t)^\circ \\ -0.5 &= \sin(6t)^\circ\end{aligned}$$

Interpolation shows that  $\sin 210^\circ = -0.5$ .

The basket will be 0.5 m under water for the first time at 35 s.

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## (E) Examples (with Technology)

- Solve the equation  $3\sin(x) - 2 = 0$

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## (E) Examples

- Solve the equation  $3\sin(x) - 2 = 0$
- The algebraic solution would be as follows:
- We can set it up as  $\sin(x) = 2/3$  so  $x = \sin^{-1}(2/3)$  giving us  $41.8^\circ$  (and the second angle being  $180^\circ - 41.8^\circ = 138.2^\circ$ )
- Note that the ratio  $2/3$  is not one of our standard ratios corresponding to our "standard" angles (30,45,60), so we would use a calculator to actually find the related acute angle of  $41.8^\circ$

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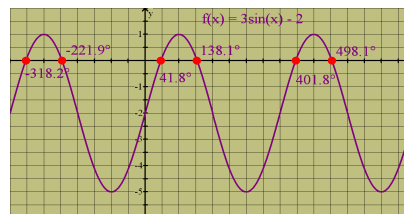
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## (E) Examples

- We can now solve the equation  $3\sin(x) - 2 = 0$  by graphing  $f(x) = 3\sin(x) - 2$  and looking for the x-intercepts



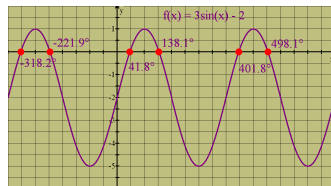
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## (E) Examples



- Notice that there are 2 solutions within the limited domain of  $0^\circ \leq \theta \leq 360^\circ$
- However, if we expand our domain, then we get two new solutions for every additional period we add
- The new solutions are related to the original solutions, as they represent the positive and negative co-terminal angles
- We can determine their values by simply adding or subtracting multiples of  $360^\circ$  (the period of the given function)

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## (E) Examples

- Solve the following equations:

$$6\sin \theta + 4 = 0 \quad \text{for } -\pi \leq \theta \leq \pi$$

$$4\sin 2\theta = 7\cos \theta \quad \text{for } -1.5 \leq \theta \leq 3$$

$$2\sin x - 4\sin^2 x = 2 \tan\left(\frac{x}{2}\right) \quad \text{for } -8 \leq \theta \leq 0$$

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## (F) Solving Equations with Technology

- The monthly sales of lawn equipment can be modelled by the following function, where  $S$  is the monthly sales in thousands of units and  $t$  is the time in months,  $t = 1$  corresponds to January.

$$S(t) = 32.4 \sin\left(\frac{\pi}{6}t\right) + 53.5$$

- (a) How many units will be sold in August?
- (b) In which month will 70 000 units be sold?
- (c) According to this model, how many times will the company sell 70 000 units over the next ten years?

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## (C) Internet Links

- [Introductory Exercises from U. of Sask EMR](#) → try introductory questions first, but skip those involving proving identities
- [Solving Trigonometric Equations - on-line math lesson from MathTV](#)
- [Trigonometric Equations and The Unit Circle from AnalyzeMath](#)

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## (D) Homework

- HH Textbook
- 13F2, Q1abcdgi
- 13F3, 2aefghi, 4ab
- 13H, 3abcd, 4acdefg