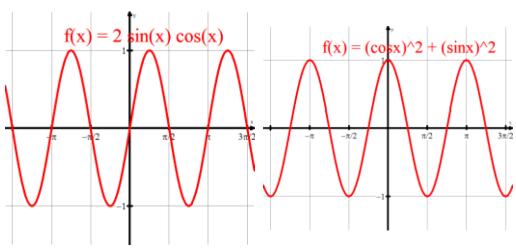
T.3.3 – Trigonometric Identities – Double Angle Formulas

IB Math SL - Santowski

Fast Five

- Graph the following functions and develop an alternative equation for the graphed function (i.e. Develop an identity for the given functions)
- f(x) = 2sin(x)cos(x)
- g(x) = $\cos^2(x) - \sin^2(x)$

Fast Five



(A) Intro

- To continue working with trig identities, we will introduce the DOUBLE ANGLE formulas WITHOUT any attempt at a proof.
- To see a proof based upon the SUM & DIFFERENCE formulas, see:
- (a) The Math Page
- To see videos on DOUBLE ANGLE FORMULAS:
- (a) http://www.youtube.com/watch?v=VzKUIr-s5O0&feature=related
- (b) http://www.youtube.com/watch?v=FAtQrZegxfI&feature=related
- (c) http://www.youtube.com/watch?v=B8psoIRChkM&feature=related

(B) Double Angle Formulas

- sin(2x) = 2 sin(x) cos(x)
- cos(2x) = $\cos^2(x) - \sin^2(x)$

(B) Double Angle Formulas

- Working with cos(2x) = $\cos^2(x) - \sin^2(x)$
- But recall the Pythagorean Identity where $\sin^2(x) + \cos^2(x) = 1$
- So $\sin^2(x) = 1 - \cos^2(x)$
- And $\cos^2(x) = 1 - \sin^2(x)$
- So $\cos(2x) = \cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$
- And $\cos(2x) = (1 - \sin^2(x)) - \sin^2(x) = 1 - 2\sin^2(x)$

(B) Double Angle Formulas

- Working with cos(2x)

- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\cos(2x) = 2\cos^2(x) - 1$
- $\cos(2x) = 1 - 2\sin^2(x)$

(B) Double Angle Formulas

- $\cos(2x) = 2\cos^2(x) - 1$
- So we can get into "power reducing" formulas for $\cos^2 x$ ➔ ½(1 + cos(2x))

- $\cos(2x) = 1 - 2\sin^2(x)$
- So we can get into "power reducing" formulas for $\sin^2 x$ ➔ ½(1 - cos(2x))

(C) Double Angle Identities - Applications

- (A) Simplifying applications
- (B) Power reducing applications
- (C) Trig equations applications

(C) Double Angle Identities - Applications

- (a) Determine the value of sin(2x) and cos(2x) if

$$\sin(\theta) = \frac{1}{4} \quad \text{and} \quad \frac{\pi}{2} \le \theta \le \pi$$

- (b) Determine the values of sin(2x) and cos(2x) if

$$\cos(\theta) = -\frac{\sqrt{5}}{6} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$

- (c) Determine the values of sin(2x) and cos(2x) if

$$\tan(\theta) = \frac{2}{5} \quad \text{and} \quad \pi \le \theta \le \frac{3\pi}{2}$$

(C) Double Angle Identities - Applications

- Simplify the following expressions:

(a) $\dfrac{\sin 2x}{\cos x}$ (b) $\cos(2x)+1$

(c) $\dfrac{\cos x \sin 2x}{1+\cos 2x}$ (d) $\cos(2x)+2\sin^2(x)+1$

(e) $\dfrac{\cos 2x}{\cos x+\sin x}$ (f) $\dfrac{\cos 2x}{\cos x-\sin x}-\sin x$

(C) Double Angle Identities - Applications

- Write as a single function:

(a) $\sin x \cos x$

(b) $2\cos^2 x - 2\sin^2 x$

(c) $2\cos^2 x - \sin^2 x$

(C) Double Angle Identities - Applications

- Use the double angle identities to prove that the following equations are identities:

$$\text{(a)} \quad \frac{\sin 2x}{1+\cos 2x} = \tan x$$

$$\text{(b)} \quad \frac{1-\sin 2x}{\cos 2x} = \frac{\cos 2x}{1+\sin 2x}$$

(C) Double Angle Identities - Applications

- Solve the following equations, making use of the double angle formulas for key substitutions:

(a) $\cos 2x = \cos^2 x \quad \text{for } -\pi \le x \le \pi$

(b) $\sin 2x = \cos x \quad \text{for } -\pi \le 2x \le \pi$

(c) $\sin 2x + \sin x = 0 \quad \text{for } 0 \le x \le 2\pi$

(d) $\cos 2x + \cos x = 0 \quad \text{for } 0 \le x \le 2\pi$

(e) $\cos^2 x - 2\sin x \cos x - \sin^2 x = 0 \quad \text{for } 0 \le 2x \le \pi$

(C) Double Angle Identities - Applications

- Use the idea of "power reduction" to rewrite an equivalent expression for:

(a) $y = \sin^4 x$

(b) $y = \cos^4 x$

Homework

- HH Text
- EX 13J, Q1-8, page 293