T25 – Algebra of Quadratics – Completing the Square

IB Math SL1 - Santowski

Fast Five

- Find the range of the parabola $y = -2(x – 4)(x + R)$
- Find the minimum point of $y = x^2 – bx + 4$
- Given the equation $4 + 7 = 11$
- Identify which properties of real numbers are highlighted by the following statements:
- (1) $4 + 7 + 0 = 11$
- (2) $4 + 7 + 3 – 3 = 11$

Fast Five

- (1) the axis of symmetry is $x = 0.5(-4R) = -2R$
- Therefore $f(-2R) = -2(-2R – 4)(-2R + R) = (4R + 8)(R)$
- So the vertex is $(-2R, 4R^2 + 8R)$ making the range $y \leq 4R^2 + 8R$
- (2) the axis of symmetry of $y = x^2 – bx + 4$ is $x = b/2$, so $f(b/2) = b^2/4 – b(b/2) + 4 = 4 - b^2/4$
- So the minimum point is $(b/2, 4 - b^2/4)$

Lesson Objectives

- Understand the rationale behind the completing the square technique: converting from standard form to vertex form
- Review the completing the square method for the equation/expression $f(x) = ax^2 + bx + c$ when *a*=1 and when *a* in not equal to 1
- Explain the graphic significance of the vertex form of the eqn $f(x) = a(x – h)^2 + k$
- Solving Eqn (algebra/graphic connection)

(A) Review

- A perfect square is the product of something multiplied by itself, such as $25 = 5^2$.
- Recall that a perfect square trinomial is one in the form as follows:
- EXPAND:
- $(x – R)^2 = x^2 – 2Rx + R^2$
- $(x + R)^2 = x^2 + 2Rx + R^2$
- FACTOR:
- $x^2 – 2Rx + R^2 = (x – R)^2$
- $x^2 + 2Rx + R^2 = (x + R)^2$

(B) Looking for Patterns

- Expand $(x + 10)^2$ using FOIL.
- Write in words the three steps you take to expand a binomial squared.
- 1) to get the first term of the quadratic:
- 2) to get the second term of the quadratic:
- 3) to get the third/last term of the quadratic:

(B) Looking for Patterns

- Consider the following equivalent forms (factored & expanded) ➔ what patterns do we see?

Factored form (binomial squared)	Expanded form (trinomial)
$(x + 1)^2$	$x^2 + 2x + 1$
$(x - 2)^2$	$x^2 - 4x + 4$
$(x + 3)^2$	$x^2 + 6x + 9$
$(x - 4)^2$	$x^2 - 8x + 16$
$(x - 5)^2$	$x^2 - 10x + 25$
$(x + 6)^2$	$x^2 + 12x + 36$

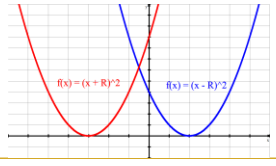
(B) Looking for Patterns

- Expand the following:

$(x - 2n)^2$	
$(x + h)^2$	
$(x - b/2a)^2$	

(C) Graphic Significance of Perfect Square Trinomials

- Given the quadratic $f(x) = (x \pm R)^2$
- or $f(x) = x^2 \pm 2Rx + R^2$, we see the following graph:

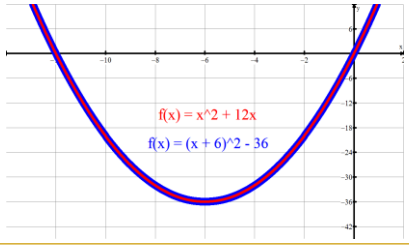


(D) Completing the Square Technique

- The phrase "completing the square" refers to the sequence of steps performed on a quadratic expression in order to write it in **the different but equivalent form** of the square of a binomial.
- For example: $x^2 + 12x = x^2 + 12x$ **+36-36**$=(x + 6)^2 - 36$
- The choice to add/subtract the number 36 is based on the pattern you have discovered on previous slides.

(D) Completing the Square Technique

- Are the 2 equations equivalent?



(E) C/S ➔ Steps Involved

- Example: Complete the square on $x^2 + 12x + 5$
- 1. Isolate the $x^2 + bx$ terms
- 2. Take ½ of b/a, square it, and add and subtract it within the parentheses:
- 3. Factor the 1st three terms in the parentheses and distribute the a over the 4th term:
- 4. Simplify the constant term:

- Example: Complete the square on $x^2 + 12x + 5$
- $(x^2 + 6x) + 5$
- $=(x^2 + 6x + 3^2 - 3^2) + 5$
- $= (x^2 + 6x + 9 - 9) + 5$
- $=(x^2 + 6x + 9) - (9)+ 5$
- $=(x + 3)^2 - 4$

(E) C/S ➔ Steps Involved

- Example: Complete the square on $2x^2 + 12x + 5$
- 1. Factor the coefficient of x^2:
- 2. Take ½ of b/a, square it, and add and subtract it within the parentheses:
- 3. Factor the 1st three terms in the parentheses and distribute the a over the 4th term:
- 4. Simplify the constant term:

- Example: Complete the square on $2x^2 + 12x + 5$
- $2(x^2 + 6x) + 5$
- $2(x^2 + 6x + 3^2 - 3^2) + 5$
- $= 2(x^2 + 6x + 9 - 9) + 5$
- $2(x^2 + 6x + 9) - 2(9) + 5$
- $2(x + 3)^2 - 13$

(F) Practice

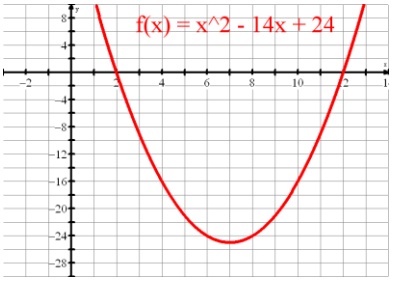
- Complete the square on each of the following. Verify by expanding. (In other words, change the form of the equation from standard to vertex form)

1. $2x^2 + 8x$
2. $-x^2 + 12x + 5$
3. $-x^2 - x - 1$
4. $3x^2 - 30x$
5. $6x^2 + 42x$

(F) Practice

- Given the quadratic function $f(x) = x^2 - 14x + 24$, change the equation to vertex form to determine the:
- (i) domain
- (ii) range
- (iii) vertex
- (iv) maximum/minimum point
- (v) maximum/minimum value
- Do you REALLY need to change the equation to find these features????

(F) Practice



(F) Practice

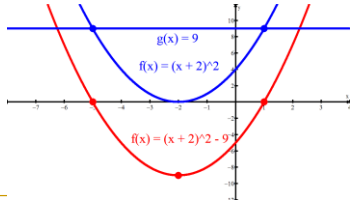
- Do you REALLY need to change $f(x) = x^2 - 14x + 25$ to find the
- (i) domain
- (ii) range
- (iii) vertex
- (iv) maximum/minimum point
- (v) maximum/minimum value
- Fair enough ➔ Find the x-intercepts of f(x)!!!

(G) Solving Using C/S

- Let's back to the basic idea of $x^2 = 9$ ➔ in other words, there exists some perfect square of 9
- Alternatively, what number(s) when squared (multiplied by itself) yields a 9?
- Clearly, the number(s) in question are +3 and -3
- What if we had the equation $(x + 2)^2 = 9$?
- Again, the expression $(x + 2)$ has two values ➔ +3 or -3
- So that $x + 2 = +3$ ➔ $x = 1$
- Or that $x + 2 = -3$ ➔ $x = -5$

(G) Solving Using C/S

- With the given equation $(x + 2)^2 = 9$, let's consider the graphical connection if I present the equations:
- (i) $0 = (x + 2)^2 - 9$
- (ii) $\begin{cases} g(x) = 9 \\ f(x) = (x+2)^2 \end{cases}$



(G) Solving Using C/S

- Solve the following equations:
- 1. $0 = 2(x - 3)^2 - 32$
- 2. $0 = -4x^2 + 10x - 3$
- 3. $-x^2 = 22x + 121$
- 4. Solve the system defined by $-x^2 = 22x + 121$
- 5. Determine the roots of $g(x) = x^2 + 22x + 100$

(H) Working with Parameters

- Given $f(x) = ax^2 + bx + c$, use the C/S method to rewrite the equation in vertex form, $f(x) = a(x - h)^2 + k$, and thereby determine h and k in terms of a,b & c
- Use the C/S method to rewrite $f(x) = ax^2 + bx + c$ in factored form, $f(x) = a(x - R_1)(x - R_2)$, and thereby determine R_1 and R_2 in terms of a,b, & c.

(I) Quadratic Modeling

- The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where h is the height in m and d is the horizontal distance of the ball in meters from the batter.
  - (a) what is the maximum height reached by the baseball?
  - (b) What is the horizontal distance of the ball from the batter when the ball reaches its maximum height?
  - (c) How far from the ground is the ball when I release the pitch?
  - (d) How high above the ground is the ball when the ball reaches the batter if she stands

(I) Quadratic Modeling

- Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for $11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every $1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?

(J) Problem Solving

- (1) If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
- (2) If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning

Homework

- HW:

- Ex 8B.2, Q1a,c,e; Q2a,c,e,f
- Ex 8C, Q1a,c,e,f; Q2a,c,e; Q3a,c
- Ex 8D.2, Q1a,c,e; Q2a,c,h; Q3a,d