

Modeling with Quadratic Functions

IB Math SL1 - Santowski

(A) Example 1

- The formula for the height, h in meters, of an object launched into the air as a function of its time in flight, t in seconds, is given by $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$
- g represents the acceleration due to gravity which is about 9.8 m/s^2 , v_0 refers to the launch velocity in m/s and h_0 represents the initial launch height in m .

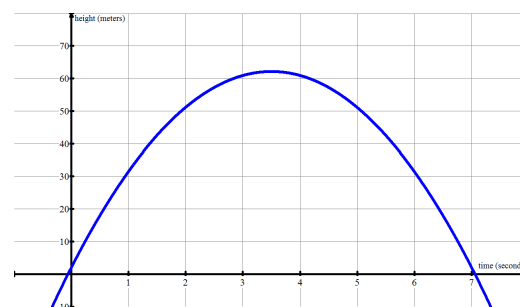
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(A) Example 1

- If a projectile has an initial velocity of 34.3 m/s and is launched 2.1 m above the ground, graphically determine:
 - (1) the equation that you will enter into the TI-84
 - (2) the time at which the projectile reaches the maximum height
 - (3) the maximum height reached by the projectile
 - (4) $h(2)$
 - (5) $h^{-1}(12)$
 - (6) state the domain and range of the relation and explain WHY
 - (7) the x-intercepts and their significance
 - (8) the total time of flight of the projectile

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(A) Example 1



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(B) Example 2

- Determine the flight time of a projectile whose height, $h(t)$ in meters, varies with time, t in seconds, as per the following formula: $h(t) = -5t^2 + 15t + 50$
- (a) Determine a reasonable domain for the function. What does it mean in context?
- (b) What is the range? What does it mean in context?
- (c) Does the projectile attain a height of 70m ?
- (d) Determine the maximum height of the projectile?
- (e) When does the object reach this height?
- (f) When does the projectile attain a height of 60 meters?

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(C) Example 3

- The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where h is the height in m and d is the horizontal distance of the ball in meters from the batter.
 - what is the maximum height reached by the baseball?
 - What is the horizontal distance of the ball from the batter when the ball reaches its maximum height?
 - How far from the ground is the ball when I release the pitch?
 - How high above the ground is the ball when the ball reaches the batter if she stands

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(D) Example 4

- The cost per hour of running a bus between Burlington and Toronto is modelled by the function $C(x) = 0.0029x^2 - 0.48x + 142$, where x is the speed of the bus in kilometres per hour, and the cost, C , is in dollars. Determine the most cost-efficient speed for the bus and the cost per hour at this speed.

(E) Example 5

- Sasha wants to build a walkway of uniform width around a rectangular flower bed that measures 20m x 30m. Her budget is \$6000 and it will cost her \$10/m² to construct the path. How wide will the walkway be?

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(E) Example 5

- Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for \$11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every \$1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?

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(F) Example 6

- (1) If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
- (2) If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning

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(G) Profit & Demand & Revenue Functions

- The demand function for a new product is $p(x) = 5x + 39$, where p represents the selling price of the product and x is the number sold in thousands. The cost function is $C(x) = 4x + 30$.
- (a) How many items must be sold for the company to break even?
- (b) What quantity of items sold will produce the maximum profit?

(G) Profit & Demand & Revenue Functions

- The demand function for a new mechanical part is $p(x) = 0.5x + 7.8$, where p is the price in dollars and x is the quantity sold in thousands. The new part can be manufactured by three different processes, A, B, or C. The cost function for each process is as follows:
 - **Process A:** $C(x) = 4.6x + 5.12$
 - **Process B:** $C(x) = 3.8x + 5.12$
 - **Process C:** $C(x) = 5.3x + 3.8$
- Use a graphing calculator to investigate the break-even quantities for each process. Which process would you recommend to the company?