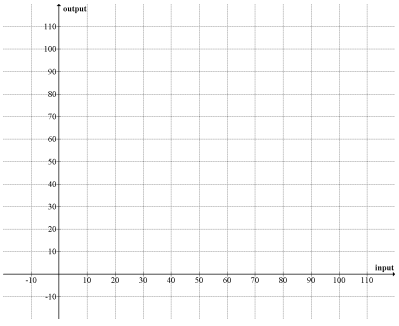
T.2.3 - The Inverse Function

Math SL1- Santowski

Opening Exercise

- On a grid, graph the points (-10,10), (0,30), (10,50), (20,70), (30,90), (40,110)
- Determine a mathematical relationship between x & y
- Prepare a mapping diagram using the same points
- Graph the line y = x on the same graph

Opening Exercise



Lesson Objectives

- Find the inverse of a function from numeric/tabular, graphic or algebraic data
- Determine whether the inverse is or is not a function
- Understand inverses as transformations
- Compose a function with its inverse to develop the identity function

(A) Inverses - The Concept

- Let's back to our input ➔ output notion for functions.
- If functions are nothing more than input/output operators, then the concept of an inverse has us considering how to go in reverse ➔ how do you get from the output back to the input?

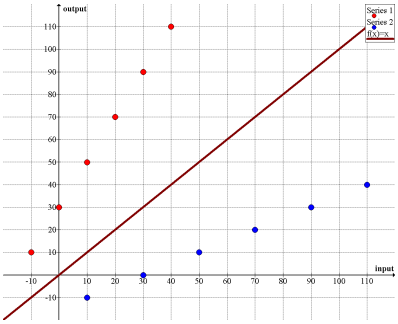
(B) Inverses - An Example of the Concept

- Consider is the idea of a relationship between °F and °C ➔ the relationship says that if we know °C we can convert to °F by means of the simplified formula double the temperature in °C and add 30°.
- If we work with this simplified formula, we can generate ordered pairs, (or tables of graphs) ➔ Order pairs would include (-10,10), (0,30), (10,50), (20,70), (30,90)
- in our input/output formula idea we would have: input ➔ times 2 ➔ add 30 ➔ output
- if we wish to discuss the idea of an inverse, we would ask "how do we go from °F back to °C?"
- So to express the inverse, we would switch or reverse the ordered pairs or our input/output notion ➔ ordered pairs are(10,-10), (30,0), (50, 10), (70, 20), (90, 30) etc….
- Our input/output formula idea would have us: do WHAT??

Opening Exercise - Revisited

- On a grid, graph the points (-10,10), (0,30), (10,50), (20,70), (30,90), (40,110)
- Determine a mathematical relationship between x & y
- Prepare a mapping diagram using the same points
- Graph the line y = x on the same graph
- Now, list the ordered pairs that are part of the "reverse" process or rather the inverse ➔ ……..
- Then graph these points of the inverse relation
- Then determine the mathematical relationship between x & y

Opening Exercise - Revisited



(C) Definition of an Inverse

- A function is a correspondence or relationship between the elements of two sets of numbers, which may be expressed as set of ordered pairs, tables, graphs or formulas.
- If the elements of the ordered pairs or mappings are reversed, the resulting set of ordered pairs or mappings are referred to as the INVERSE.
- Another point worth noting: the domain of the original function now becomes the range of the inverse; likewise, the range of the original becomes the domain of the inverse.

(D) Notation of the Inverses

- The notation used for the inverses of functions is $f^{-1}(x)$.
- IMPORTANT NOTE: $f^{-1}(x)$ does not mean $(f(x))^{-1}$ or $1 / f(x)$.

(E) Inverses as Transformations

- Place the ordered pairs on the Cartesian plane and we see the relationship between the original ordered pairs and the transformed ordered pairs of the inverse
- The relationship that exists is that the original points are reflected in the line y = x.

(F) Determining Equations of Inverses

- Let's start with the ordered pairs of the following relation:
- (-2, 1),(-1,-2), (0,-3), (1,-2), (2,1) (3,6), (4,13)
- Look for a pattern in the data ➔ can you find a mathematical relationship between x & y?
- the ordered pairs of the inverse will be
- (1,-2), (-2,-1),(-3,0), (-2,1), (1,2), (6,3), (13,4)
- So can you find a mathematical relationship between x & y in this inverse relation?
- Alternatively, can you find some algebraic method of "undoing" your mathematical relationship from before?

(F) Algebraic Method For Determining Equations

- How can I manipulate $y = x^2 - 3$ to get $y = \sqrt{(x + 3)}$, or the reverse, how can I manipulate $y = \sqrt{(x + 3)}$ to get $y = x^2 - 3$?
- Since forming the inverse involved switching the order of an ordered pairs (x↔y) then let's try this for the algebraic approach
- If $y = x^2 - 3$ → then $x = y^2 - 3$ → So $x + 3 = y^2$
- And thus $y = \sqrt{(x + 3)}$ which was the equation of our inversed ordered pairs
- Thus the algebraic method is to switch the x and y and then solve the resultant equation for y.

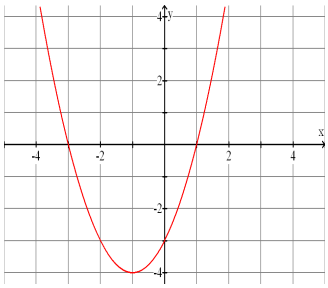
(G) Examples

- Ex. Determine the equation for the inverse of $y = 4x - 9$. Draw both graphs and find the D and R of each.
- Ex. Determine the equation for the inverse of $y = 2x^2 + 4$. Draw both and find D and R of each.
- Ex. Determine the equation for the inverse of $y = 2 - \sqrt{(x+3)}$. Draw both and find the domain and range of each.
- Ex. Determine the equation of the inverse of $y = (3x - 1)/(2x + 3)$. Draw both and find the domain and range of each.

(G) Examples

- ex 2. Consider a graph of the following data:
- 1. State do domain and range of *f*
- 2. Evaluate f(-2), f(0), $f^{-1}(1)$, $f^{-1}(-2)$
- 3. Graph the inverse
- 4. Is the inverse a function?
- 5. State the domain and range of $f^{-1}(x)$

- Here is the graph of f(x)



(G) Examples

- ex . The cost of renting a car for a day is a flat rate of $40 plus $0.10/km
- 1. Write a function r(d) to represent the total cost of a one day rental. State restrictions on domain and range.
- 2. Find the equation of the inverse. What does the equation of the inverse represent?
- 3. Give an example of how the inverse could be used?

(G) Examples

- ex . If an object is dropped from a height of 80 m, its height above the ground in meters is given by
- $h(t) = -5t^2 + 80$

- 1. Graph the function
- 2. Find and graph the inverse
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

(H) Composing with Inverses

- Let $f(x) = 2x - 7$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph

- Fold the grid/graph upon the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?

- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?
- How is this related to our graph folding exercise?

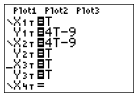
(H) Composing with Inverses

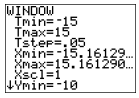
- Now let $f(x) = x^2 + 2$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph

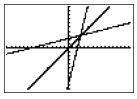
- Fold the grid/graph upon the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?

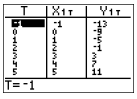
- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?
- How is this related to our graph folding exercise?

Inverses on the TI-84 & Parametric Mode

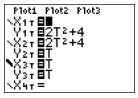


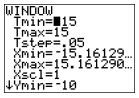


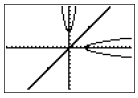


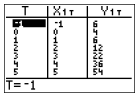


Inverses on the TI-84 & Parametric Mode









(H) Internet Links

- Inverse Function Definition - Interactive Applet from AnalyzeMath
- Inverse Function - Interactive Tutorial from AnalyzeMath
- Inverse Functions Lesson - I from PurpleMath

(H) Homework

- HW Ex 1F #1,2c, 4bc, 5, 7b, 11, 13; Ex 1G #2, 3