T2.1- Lesson 1 - Functions: Concepts and Notations

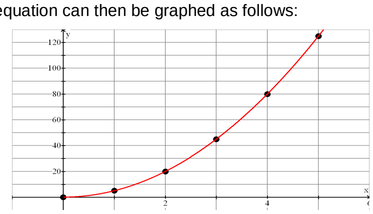
IB Math SL1 - Santowski

(A) Concept of Functions & Relations

- In many subject areas, we see relationships that exist between one quantity and another quantity.
  - ex. Galileo found that the distance an object falls is related to the time it falls.
  - ex. distance traveled in car is related to its speed.
  - ex. the amount of product you sell is related to the price you charge.
- All these relationships are classified mathematically as **Relations**.

(B) Representation of Functions & Relations

- Relations can be expressed using ordered pairs i.e. (0,0), (1,5), (2,20), (3,45), (4,80), (5,125)
- The relationships that exist between numbers are also expressed as equations: $s = 5t^2$
- This equation can then be graphed as follows:

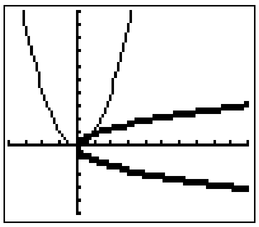


(C) Terminology of Functions & Relations

- Two terms that we use to describe the relations are **domain** and **range**.
- Domain refers to the set of all the first elements, input values, independent variable, etc.. of a relation, in this case the time. We will express domain in set notation and in interval notation
- Range refers to the set of all the second elements, output values, dependent values, etc... of the relation, in this case the distance. We will express the range in set notation and in interval notation

OCF.02.1 - Functions: Concepts and Notations

(D) Functions - The Concept



- A **function** is a special relation in which each **single** domain element corresponds to exactly **one** range element. In other words, each input value produces one unique output value
- ex. Graph the relations defined by $y = x^2$ and $x = y^2$ ➔ one is a function and one is not??

(D) Functions - The Concept

- Q? In what ways do the two graphs differ?
- In the graph of $y = x^2$, notice that each value of x has one and only one corresponding value of y.
- In the graph of $x = y^2$, notice that each value of x has two corresponding values of y.
- We therefore distinguish between the two different kinds of relations by defining one of them as a function. So a function is special relation such that each value of x has one and only one value of y.

(D) Functions - The Concept - Examples

- Make a mapping diagram for the relation {(-2,1), (-2,3), (0,3), (5,4)} and determine whether or not the relation is a function. Give a reason for your answer.
- State the domain and range of the following relation. Is the relation a function?
  {(–3, 5), (–2, 5), (–1, 5), (0, 5), (1, 5), (2, 5)}

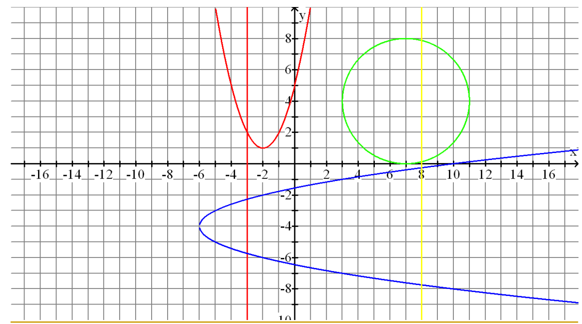
(D) Functions - The Concept - Examples

- A relation is defined by the set {(-1,2), (3,0), (5,2)}.
- (a) Sketch the set on a Cartesian plane and label the ordered pairs
- (b) Make a mapping diagram of this relation
- (c) State the domain of this relation
- (d) State the range of this relation

(E) Functions - Vertical Line Test

- To determine whether or not a relation is in fact a function, we can draw a vertical line through the graph of the relation.
- If the vertical line intersects the graph more than once, then that means the graph of the relation is not a function.
- If the vertical line intersects the graph once then the graph shows that the relation is a function.
- See the diagram on the next slide

(F) Functions - Vertical Line Test



(G) Functions - the Notation f(x)

- We have written equations in the form $y = 2x + 5$ or $y = 3x^2 - 4$.
- These equations describe the relationship between x and y, and so they describe relations ➔ since each x produced a unique y value, they are also functions
- Therefore we have another notation or method of writing these equations of functions.
- We can rewrite $y = 2x + 5$ as $f(x) = 2x + 5$ or $f : x \mapsto 2x + 5$.
- We can rewrite $y = 3x^2 - 4$ as $g(x) = 3x^2 - 4$ or $g : x \mapsto 3x^2 - 4$.

(G) Functions - the Notation f(x)

- If $y = 2x - 5$, then what function (set of mathematical instructions) do you perform on x (the input) to get y (the output)?
  - 1) double the input
  - 2) subtract 5 from result above
- Let f be the name for the set of instructions (or, the FUNCTION name) of doubling and then subtracting 5.
- Then, the function f APPLIED to x is $f(x) = 2x - 5$.
- But if $2x - 5$ is the output and f(x) is EQUAL to $2x - 5$, then f(x) must also be the output.
- Therefore, we have the ordered pairs:
  - (x,y) or
  - (x,f (x)) or
  - (x,2x - 5)

input	Output
6	2(6) – 5 = 7
-2	2(-2) – 5 =
√3	2( ) – 5 =
x	2( ) – 5 =

(H) Working with Function Notation

- For the function defined by $f(t) = 3t^2 - t + 4$, evaluate f(4):
- $f(4) = 3(4)^2 – (4) + 4 = 48 – 4 + 4 = 48$
- So notice that t = 4 is the "input" value (or the value of independent variable) and 48 is the "output" value (or the value of the dependent variable)
- So we can write f(4) = 48 or in other words, 48 (or f(4)) is the "y value" or the "y co-ordinate" on a graph
- So we would have the point (4,48) on a graph of t vs f(t)
- And as an order pair, I could write the info as (4,48), or (4, f(4)) or (4, $3(4)^2 – (4) + 4$)

(H) Working with Function Notation

- ex. For the function defined by $b(t) = 3t^2 - t + 3$, find:
- (a) b(-2) (b) b(0.5) (c) b(2)
- (d) b(t - 2) (e) $b(t^2)$ (f) b(1/x)
- ex. For the function defined by $f(x) = +\sqrt{9 - x^2}$ graph it and then find new equations and graph the following:
- (a) f(x-3) (b) f(x+2) (c) f(3x) (d) 3f(x)
- ex. For the function defined by w(a) = 4a - 6, find the value of ***a*** such that w(a) = 8

(I) Introduction to Domain and Range

- What follows in the subsequent slides are some common functions with which you must become familiar.
- Your initial investigation into these functions will be from a domain and range approach.

(B) Linear Functions

- For each function listed below, determine f(2)
- Then, graph the following functions on the TI-84 and zoom in and out to get an idea of the domain and range of each function.
- You should also check the table of values for each function to confirm the domain and range you stated after viewing the graphs.
- (i) f(x) = 2
- (ii) f(x) = -2x + 5
- (iii) f(x) = ½x – 6
- (iv) x = 2

(C) Quadratic Functions

- For each function listed below, determine g(2)
- Then graph the following functions on the TI-84 and zoom in and out to get an idea of the domain and range of each function.
- You should also check the table of values for each function to confirm the domain and range you stated after viewing the graphs.
- What seems to be the key point on a quadratic function in terms of domain and range?
- (i) $g(x) = x^2$ (ii) $g(x) = (x-3)^2 + 4$
- (iii) $g(x) = -(x – 3)^2 + 4$ (iv) g(x) = (2x – 3)(3 - x)
- (v) $g(x) = 0.25x^2 – x + 6$

(D) Root Functions

- For each function listed below, determine k(2)
- Graph the following functions on the TI-84 and zoom in and out to get an idea of the domain and range of each function
- You should also check the table of values for each function to confirm the domain and range you stated after viewing the graphs.
- What seems to be the key point on root function in terms of domain and range?
- (i) k(x) = sqr(x) or $\sqrt{(x)}$
- (ii) $k(x) = \sqrt{(x – 2)} + 4$
- (iii) $k(x) = - \sqrt{(x + 1)} – 3$
- (iv) $k(x) = 2 \sqrt{(3x)} + 1$

(E) Absolute Value Functions

- For each function listed below, determine h(2)
- Graph the following functions on the TI-84 and zoom in and out to get an idea of the domain and range of each function
- You should also check the table of values for each function to confirm the domain and range you stated after viewing the graphs.
- What seems to be the key point on an absolute value function in terms of domain and range?
- (i) h(x) = | x | (ii) h(x) = | x – 2 | + 4
- (iii) h(x) = - | x + 1 | - 3 (iv) h(x) = 2| 3x | + 1

(F) Reciprocal Functions

- For each function listed below, determine m(2)
- Graph the following functions on the TI-84 and zoom in and out to get an idea of the domain and range of each function.
- You should also check the table of values for each function to confirm the domain and range you stated after viewing the graphs.
- What seems to be the key point on a reciprocal function in terms of domain and range?
- (i) m(x) = 1/x (ii) m(x) = -1/x
- (iii) m(x) = 1/(x – 2) + 4 (iv) m(x) = - 1/(x + 1) – 3
- (v) m(x) = 2 [1/(3x)] + 1

(G) Summary

- Summarize your findings as you make a generalization about the domains and ranges of:

- (1) Linear Functions
- (2) Quadratic Functions
- (3) Root Functions
- (4) Absolute value Functions
- (5) Reciprocal Functions

- Now that you have seen some examples, go to the following link and work through the following on-line examples: Domains of Functions from Visual Calculus

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(I) Internet Links

- College Algebra Tutorial on Introduction to Functions - West Texas A&M
- College Algebra Tutorial on Graphs of Functions Part I - from West Texas A&M
- Functions Lesson - I from PurpleMath
- Functions Lesson - Domain and Range from PurpleMath
- Functions from Visual Calculus

(J) Homework

- HW

- Ex 1A #1ce,2bcdf;
- Ex 1B #1acd, 2ad;
- Ex 1C #2d, 3b, 4e,5a, 12, 14;
- Ex 1D #2, 3