

# Composition of Functions

Math SL1 - Santowski

## (A) Review

- Given the functions  $f(x)$  and  $g(x)$  with their specified domains, we can create new functions by adding, subtracting, multiplying, dividing, inverting, and reciprocating the initial functions, once again over a new, modified domain
- We have another way of creating new functions → we can **compose** two functions which is basically a **substitution of one function into another**.

## (B) Composition of Functions – An Example

- The following examples will be various ways of representing the composition of functions
- ex 1. Andrew earns a daily wage of \$20/h plus \$15/d for travel expenses.
- We can write his daily earnings as an equation as  $Earnings = 20h + 15$ .
- However, Andrew also pays union fees at 2.5% of his daily earnings, which we can write as the equation  $Fees = 0.025 \times (\text{daily earnings})$
- We can demonstrate with a table of values

Hours worked per day	Daily Earnings	Union Fees Paid
2	$20(2) + 15 = 55$	$0.025(55) = 1.375$
5	$20(5) + 15 = 115$	$0.025(115) = 2.875$

## (B) Composition of Functions – An Example

- What we see is that the one function value (daily earnings or E) is being substituted into the second function ( $Fees = 0.025 \times \text{daily earnings}$ ) in order to generate the value for the union fees.
- We can generate a direct formula for the union fees by substituting the earnings function into the Fees function as follows:  $Fees = 0.025(20h + 15)$ .
- Hence, the Fees function is called a composed function as  $Fees(\text{daily earnings}) = 0.025 \times \text{daily earnings}$

### (C) Composition of Functions – Example #2

- ex 2. We will now define  $f$  and  $g$  as follows:
- $f = \{(3,2), (5,1), (7,4), (9,3), (11,5)\}$
- $g = \{(1,3), (2,5), (3,7), (4,9), (5,10)\}$
- We will now compose these two functions as follows:
- (i) we will substitute  $g$  into  $f$  which we will notate in two ways:  $f(g(x))$  or as  $f \circ g(x)$ .
- We will evaluate  $f \circ g(3)$  (or  $f(g(3))$ ) → ?????

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### (C) Composition of Functions – Example #2

- ex 2. We will now define  $f$  and  $g$  as follows:
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- $g = \{(1,3), (2,5), (3,7), (4,9), (5,10)\}$
- We will now compose these two functions as follows:
- (i) we will substitute  $g$  into  $f$  which we will notate in two ways:  $f(g(x))$  or as  $f \circ g(x)$ .
- We will evaluate  $f \circ g(3)$  (or  $f(g(3))$ ) → we will substitute a specific  $g$  function value into  $f$ , that of  $g(3)$ .
- The  $g$  function value at  $x=3$  is 7 (i.e.  $g(3) = 7$ )
- So now we evaluate  $f$  at the new value of  $x=7$  → we look at our  $f$  data and see that  $f(7) = 4$
- (ii) evaluate  $f \circ g(1)$
- (iii) evaluate  $f \circ g(5)$  and see what happens → why?
- (iv) evaluate  $g \circ f(9)$  and  $g(f(7))$  and  $g \circ g(1)$

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### (C) Composition of Functions – Example #3

- We can define  $f$  and  $g$  differently, this time as formulas:
- $f(x) = x^2 - 3$
- $g(x) = 2x + 7$
- We will try the following:
- (i)  $f(g(3))$  or  $f \circ g(3)$
- (ii)  $g \circ f(3)$  or  $g(f(3))$
- (ii)  $f \circ g(x)$  and  $g \circ f(x)$
- (ii) evaluate  $f \circ g(5)$
- (iii) evaluate  $g \circ f(9)$  and  $g(f(7))$  and  $g \circ g(1)$
- Repeat if  $f(x) = \log_2(x)$  and  $g(x) = 2^{2x}$

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### (D) Composition of Functions – New Domains

- When we create a new function from 2 or more functions (as in composing 2 functions), we must consider the domain of the new function
- So domain restrictions to remember from previous courses:
- Square root functions → we take square roots of positive numbers
- Reciprocal functions → denominators cannot be zero
- Logarithmic functions → you cannot take the log of a negative value
- Exponential, polynomial, absolute value, trigonometric functions → unrestricted domains
- Now go back to the previous slide and state the domain of the new functions you created

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## (E) Internet Links

- [Composition of Functions from Visual Calculus](#) → Read the Discussion
- [Tutorial on Composition of Functions from AnalyzeMath](#)
- [Composition of Functions Lesson - From PurpleMath](#)
- [Operations with Functions from WTAMU](#) → includes examples with composition

## (F) Classwork

- Now go to the following link for our classwork:
- [Composition of Functions Questions from AnalyzeMath](#)

## (G) Homework

- Textbook → Ex 1D #2, 3