T1.1 – Lesson 3 - Arithmetic & Geometric Series & Summation Notation

Math SL1 - Santowski

Fast Five

- Find the sum of the first 100 numbers
- Outline a way to solve this problem and then carry out your plan

- Find the sum of the first 25 perfect squares
- Outline a way to solve this problem and then carry out your plan

(A) Review

- A sequence is a set of ordered terms, possibly related by some pattern
- One such pattern is called arithmetic because each pair of consecutive terms has a common difference
- The general term of an arithmetic sequence is defined by the formula $u_n = u_1 + (n - 1)d$
- A geometric sequence is one in which the consecutive terms differ by a common ratio
- The general term of an geometric sequence is defined by the formula $u_n = u_1 \times r^{(n - 1)}$

(B) Arithmetic Series

- A **series** is defined as the sum of the terms of a sequence.
- As an example, start by finding the sum of the first 100 numbers and showing an easy way to set it up:

S_{100}	1	2	3	4	5	....	99	100
S_{100}	100	99	98	97	96	......	2	1
$2S_{100}$	101	101	101	101	101	......	101	101

- So then the sum is (101)(100)÷2

(B) Arithmetic Series

- For an arithmetic sequence then the formula for the sum of its terms is:

$$S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}[2a + (n-1)d]$$

(C) Examples

- Ex 1. Find the sum of the series13 + 24 + 35 + … + 156
- Ex 2. For the series 2 + 11 + 20 + 29 + ..., find u_{20} and S_{20}
- Ex 3. The fifth term of an arithmetic series is 9 and the sum of the first 16 is 480. Find the first three terms of the series.
- Ex 4. In an arithmetic series of 50 terms, the 17th term is 53 and the 28th term is 86. Find the sum of the series.

(C) Examples

- ex 5. Shayla deposits $128 into her account. Each week she deposits $7 less than the previous week until she deposits her last deposit of $2. What total amount did she deposit?
- ex 6. Jayne buys 10 widgets on the Jan 1st , 15 on the 1st of Feb, 20 on the 1st of March, etc..... How many widgets has she acquired in 2 years? How long does it take her to acquire 5,000 widgets?

(D) Summation Notation

- Summation notation is a shorthand way of saying take the sum of certain terms of a sequence ➔ the Greek letter sigma, Σ is used to indicate a summation
- In the expression $\sum_{i=1}^{n} a_i$
- *i* represents the term number (or index of summation), and a_i represents the general term of the sequence being summed
- So therefore, $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + ... + a_n$

(E) Summation Notation

- Ex 1 – Arithmetic Series

$$\sum_{n=1}^{6}(n+1)=(1+1)+(2+1)+(3+1)+(4+1)+(5+1)+(6+1)$$
$$\sum_{n=1}^{6}(n+1)=(2)+(3)+(4)+(5)+(6)+(7)$$
$$\sum_{n=1}^{6}(n+1)=27$$

Lesson 2 Fast Five

- Determine S_{17} = 1 + 3 + 9 + 27 + 81 + 243 + …..
- NO CALCULATOR

(E) Examples of Summation Notation

- Write out the series expansion for the following and then use the TI-84 to evaluate the sums:

$$\sum_{i=1}^{8} i^2 =$$
$$\sum_{i=1}^{10}\left(\frac{i-4}{i}\right)=$$
$$\sum_{i=4}^{9} e^{\ln(i)} =$$

(E) Examples of Summation Notation

- Evaluate the series that are defined in the following summation notations:
- $S_6=\sum_{i=1}^{6}(3i+5)=$

  $S_{20}=\sum_{i=1}^{20}(5)=$

  $S_{25}=\sum_{i=1}^{25}\left(11-\frac{1}{2}i\right)=$

  $S=\sum_{i=6}^{10}(i+5)=$

  is $\sum_{i=1}^{5}(4i)=4\times\sum_{i=1}^{5}(i)$

(F) Geometric Series

- Find the sum of the first 7 terms of the series
- S_7 = 1 + 3 + 9 + 27 + 81 + 243 + 729
- $S_7 = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + ar^6$

S_7	1	3	3^2	3^3	3^4	3^5	3^6	
$3S_7$		3	3^2	3^3	3^4	3^5	3^6	3^7

- $3S_7 - S_7 = 2S_7 = (3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7) - (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6) = 3^7 - 1$
- $S_7 = ½(3^7 - 1)$

(F) Geometric Series

- So in general, the formula for the sum of a geometric series is:

$$S_n = \frac{(u_{n+1} - u_1)}{r-1} = \frac{u_1(r^n - 1)}{r-1}$$

(G) Examples

- ex 1. Find S_8 given and rewrite in summation notation:
- (a) 2 - 6 + 18 - 54 + ...
- (b) 200 + 100 + 50 + 25 + ...
- ex 2. Find the total amount you make if you were paid a rupee a day, but the amount was doubled every day for a month
- ex 3. Find the sum 1/16 + 1/4 + 1 + 4 + ... + 65536

(G) Examples

- Ex 4. The fifth term of a geometric series is 405 and the sixth term is 1215. Find the sum of the first nine terms.
- ex 5. A ball drops from a height of 16 m and its height on the bounce is 5/8th of the previous maximum height. Determine the total height bounced by the ball after it touches the ground for the 7th bounce.

(H) Examples of Summation Notation

- Ex 2 Geometric Series

$$\sum_{n=1}^{4} 4(-\frac{1}{2})^{n-1} = (4)(-0.5)^0 + (4)(-0.5)^1 + (4)(-0.5)^2 + (4)(-0.5)^3$$

$$\sum_{n=1}^{4} 4(-\frac{1}{2})^{n-1} = (4)(1) + (4)(-0.5) + (4)(0.25) + (4)(-0.125)$$

$$\sum_{n=1}^{4} (4)(-\frac{1}{2})^{n-1} = 4 + (-2) + 1 + (-0.5) = 2.5$$

(H) Examples of Summation Notation

- Given the series S = 200 + 100 + 50 + 25 …

- (i) Write a summation expression for the series
- (ii) Determine S_6
- (iii) Determine S_{15}
- (iv) Determine S_{21}
- (v) Predict $S_{1,000,000}$

(H) Examples of Summation Notation

- The series ½ + ¼ + 1/8 + 1/16 + ... is an example of an infinite geometric series.

- (a) Determine the sum of this series.
- (b) Is it possible to find the sum of *any infinite geometric sequence? Explain.*
- (c) Under what conditions is it possible to find the sum of an infinite geometric sequence

(H) Examples of Summation Notation

- Show that the sum of *n terms of the series*
- *2 + 1 + ½ + ¼ + ... u_n is always* less than 4, where *n is any natural number.*

- *Explain what the following notations mean:*

$$S_6 = \sum_{i=1}^{6} \left(2^{2-i}\right)$$

$$S_n = \sum_{i=1}^{n} \left(2^{2-i}\right)$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(2^{2-i}\right) =$$

(I) Internet Links

- Geometric Sequences & Series From West Texas A&M
- Arithmetic Sequences & Series From West Texas A&M U

(J) Homework

- HW:
- Ex 2E.1 #1ae;
- Ex 2E.2 #1c, 2a, 3,5, 6, 11;
- Ex 2E.3 #1bc, 2cd, 4, 6, 7
- HW Ex 2F #1c, 3c, 4c, 5ab and
- IB packet #1 - 8