Lesson 58 – Normal Distributions

IB Math SL1 - Santowski

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Introduction to the "Normal Distribution"

- Run through the "coin tossing" simulation from textbook on p472 and the "dice rolling" simulation

- KEY point to make ➔ if we run an "experiment" enough times (i.e collect sufficient data), then our histograms (or our distribution "curves") start taking on a consistent "shape" ➔ this shape will be referred to as the "**normal distribution**"

Introduction to the "Normal Distribution"

- Since our distribution came from data we collected, we can analyze the data for key statistical features ➔ mean and standard deviation (as well as others ➔ Q_1, Q_3, IQR, median)

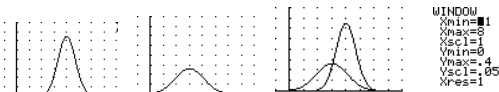- We will discuss mean and standard deviation as we analyze our normal distributions

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The Normal Distribution

- The Normal distribution has the shape of a "bell curve" with parameters μ and σ^2 that determine the center and spread:
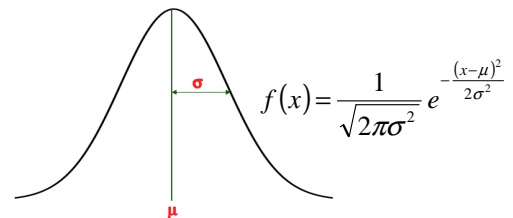
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
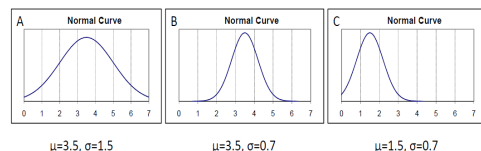

The Normal Distribution - Example

- Here are scores that IB SL1 students scored on Year 1 June exams where the mean was 4.0 and the standard deviation was 0.7

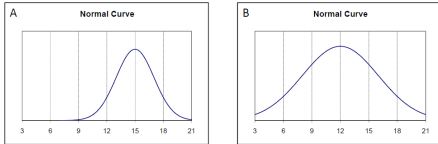- Last year, the mean was 3.0 with a standard deviation of 1.1

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WINDOW
Xmin=█1
Xmax=8
Xscl=1
Ymin=0
Ymax=.4
Yscl=.05
Xres=1
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The Normal Distribution

A Normal Curve	B Normal Curve	C Normal Curve
$\mu=3.5$, $\sigma=1.5$	$\mu=3.5$, $\sigma=0.7$	$\mu=1.5$, $\sigma=0.7$

- Curve A and Curve B have the same mean,
- Curve B and Curve C have the same standard deviation,
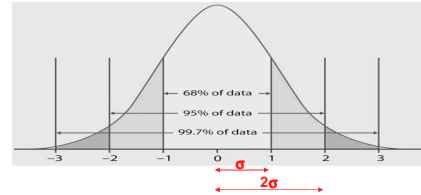- Each Curve has a total area of 1.

The Normal Distribution

- Which normal curve has a greater mean?
- Which normal curve has a greater standard deviation?

Property of Normal Distributions

- Normal distribution follows the *68-95-99.7 rule*:
  - 68% of observations are between $\mu - \sigma$ and $\mu + \sigma$
  - 95% of observations are between $\mu - 2\sigma$ and $\mu + 2\sigma$
  - 99.7% of observations are between $\mu - 3\sigma$ and $\mu + 3\sigma$
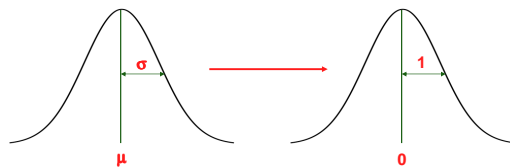

The Normal Distribution - Example

- Here are scores that IB SL1 students scored on Year 1 June exams where the mean was 4.0 and the standard deviation was 0.7

- (a) what percentage of students scored between 3.3 and 4.7?
- (b) What percentage of students scored between 2.6 and 4.7?
- (c) What percentage of students scored between 1.9 and 5.4?
- (d) If a passing grade was set at 2.6, what percentage of students passed the exam?
- (e) If Honors designations were given to students who scored over 5.4, what percentage of students were given an honors designation?

Standardization

- If we only have a standard normal table, then we need to *transform* our non-standard normal distribution into a standard one
  - This process is called **standardization**

Standardization Formula

- We convert a non-standard normal distribution into a standard normal distribution using a *linear transformation*
- If X has a $N(\mu, \sigma^2)$ distribution, then we can convert to Z which follows a $N(0,1)$ distribution

$$Z = (X-\mu)/\sigma$$

- First, subtract the mean μ from X
- Then, divide by the standard deviation σ of X

2

Homework

- HW

- Ex 29G.1 #1, 3, 6;
- Ex 29G.2 (using GDC) #1, 2abc;