# L57 – Expected Values

IB Math SL1 - Santowski

## (F) Expected Values

- Example ➔ a single die ➔ You roll a die 240 times. How many 3's to you EXPECT to roll?
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- (i.e. Determine the expectation of rolling a 3 if you roll a die 240 times)
- ANS ➔ 1/6 x 240 = 40 ➔ implies the formula of (n)x(p)
- BUT remember our focus now is not upon a single event (rolling a 3) but ALL possible outcomes and the resultant distribution of outcomes ➔ so .....

## (F) Expected Values

- The **mean** of a random variable ➔ a measure of central tendency ➔ also known as its **expected value**,E(x), is **weighted average** of all the values that a random variable would assume in the long run.

## (F) Expected Value

- So back to the die ➔ what is the expected value when the die is rolled?
- Our "weighted average" is determined by sum of the products of outcomes and their probabilities $E(X) = \sum_i x_i \times p(x_i)$

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- X = {1,2,3,4,5,6}
- $p(x_i) = 1/6$
- E(X) = (1)(1/6) + (2)(1/6) + (3)(1/6) + (4)(1/6) + (5)(1/6) + (6)(1/6)
- E(X) = 21/6 or 3.5

## (F) Expected Value

- E(x) is <u>not</u> the value of the random variable x that you "expect" to observe if you perform the experiment **<u>once</u>**
- E(x) is a "**<u>long run</u>**" average; if you perform the experiment many times and observe the random variable x each time, then the average x of these observed x-values will get closer to E(x) as you observe more and more values of the random variable x.

## (F) Expected Value

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- X = number of heads = {0,1,2}
- p(0)=1/4, p(1)=1/2, p(2)=1/4
- Weighted average = 0*1/4 + 1*1/2 + 2*1/4 = 1

## (F) Expected Value

- Expectations can be used to describe the potential gains and losses from games.
- Ex. Roll a die. If the side that comes up is odd, you win the $ equivalent of that side. If it is even, you lose $4.
- Ex. Lottery – You pick 3 different numbers between 1 and 12. If you pick all the numbers correctly you win $100. What are your expected earnings if it costs $1 to play?

## (F) Expected Value

- Ex. Roll a die. If the side that comes up is odd, you win the $ equivalent of that side. If it is even, you lose $4.
- Let X = your earnings
- X=1 P(X=1) = P({1}) =1/6
- X=3 P(X=1) = P({3}) =1/6
- X=5 P(X=1) = P({5}) =1/6
- X=-4 P(X=1) = P({2,4,6}) =3/6
- E(X) = 1*1/6 + 3*1/6 + 5*1/6 + (-4)*1/2
- E(X) = 1/6 + 3/6 +5/6 – 2= -1/2

## (F) Expected Value

- Ex. Lottery – You pick 3 different numbers between 1 and 12. If you pick all the numbers correctly you win $100. What are your expected earnings if it costs $1 to play?

- Let X = your earnings
- X = 100-1 = 99
- X = -1

- P(X=99) = 1/(12 3) = 1/220
- P(X=-1) = 1-1/220 = 219/220
- E(X) = 100*1/220 + (-1)*219/220 = -119/220 = -0.54

## (F) Expected Value

- For example, an American roulette wheel has 38 places where the ball may land, all equally likely.

- A winning bet on a single number pays 35-to-1, meaning that the original stake is not lost, and 35 times that amount is won, so you receive 36 times what you've bet.

- Considering all 38 possible outcomes, Determine the expected value of the profit resulting from a dollar bet on a single number

## (F) Expected Value

- The net change in your financial holdings is −$1 when you lose, and $35 when you win, so your expected winnings are.....

- Outcomes are X = -$1 and X = +$35
- So E(X) = (-1)(37/38) + 35(1/38) = -0.0526

- Thus one may expect, on average, to lose about five cents for every dollar bet, and the **expected value** of a one-dollar bet is $0.9474.

- In gambling, an event of which the expected value equals the stake (i.e. the better's expected profit, or net gain, is zero) is called a "fair game".

## (F) Expected Value

- The concept of Expected Value can be used to describe the expected monetary returns

- An investment in Project A will result in a ***loss*** of $26,000 with probability 0.30, break even with probability 0.50, or result in a profit of $68,000 with probability 0.20.
- An investment in Project B will result in a ***loss*** of $71,000 with probability 0.20, break even with probability 0.65, or result in a profit of $143,000 with probability 0.15.
- Which investment is better?

## Tools to calculate E(X)-Project A

- Random Variable (X)- The amount of money received from the investment in Project A
- X can assume only $x_1$ , $x_2$ , $x_3$

- X= $x_1$ is the event that we have Loss
- X= $x_2$ is the event that we are breaking even
- X= $x_3$ is the event that we have a Profit

- $x_1$=$-26,000
- $x_2$=$0
- $x_3$=$68,000

- P(X= $x_1$)=0.3
- P(X= $x_2$)= 0.5
- P(X= $x_3$)= 0.2

## Tools to calculate E(X)-Project B

- Random Variable (X)- The amount of money received from the investment in Project B
- X can assume only $x_1$ , $x_2$ , $x_3$

- X= $x_1$ is the event that we have Loss
- X= $x_2$ is the event that we are breaking even
- X= $x_3$ is the event that we have a Profit

- $x_1$=$-71,000
- $x_2$=$0
- $x_3$=$143,000

- P(X= $x_1$)=0.2
- P(X= $x_2$)= 0.65
- P(X= $x_3$)= 0.15

## Tools to calculate E(X)-Project A & B

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Project A:

$$E(X) = 0.30 \cdot (-\$26{,}000) + 0.50 \cdot \$0 + 0.20 \cdot \$68{,}000$$
$$= \$5800$$

Project B:

$$E(X) = 0.20 \cdot (-\$71{,}000) + 0.65 \cdot \$0 + 0.15 \cdot \$143{,}000$$
$$= \$7250$$

## Homework

- HW
- Ex 29C, p716, Q10-14
- Ex 29D, p720, Q1,2,3,5,6,7 (mean only)