

## CALCULATOR ACTIVE SECTION

PART A - MULTIPLE CHOICE

1. In September 1998 the population of the country West Goma in millions was modeled by  $f(x) = 16.6e^{0.0019x}$ . At the same time, the population of East Goma in millions was modeled by  $f(x) = 14.6e^{0.0124x}$ . In both formulas,  $x$  is the year and  $x = 0$  corresponds to September 1998. Assuming that the trends continue, when to the nearest year will the two countries have equal populations?

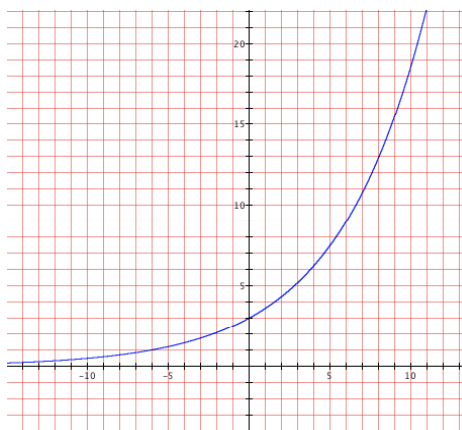
A) 12                                      B) 2010                                      C) 2007                                      D) 1986

2. State whether  $f(x) = 3.4(0.8)^x$  is an example of exponential growth or decay, and state the constant percentage rate of growth or decay.

A) Exponential Growth; 80%                                      C) Exponential Growth; 20%  
 B) Exponential Decay; 80%                                      D) Exponential Decay; 20%

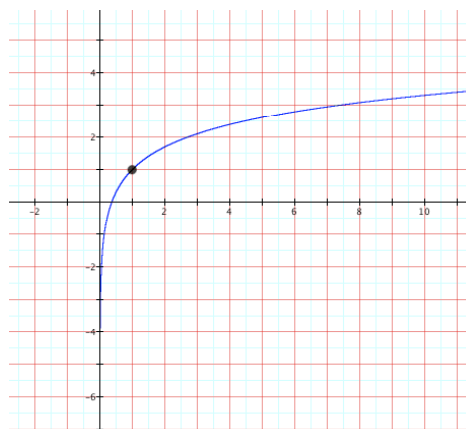
3. Which equation is the correct equation for the graph below?

- A)  $y = 3(1.2)^x$   
 B)  $y = 3(0.8)^x$   
 C)  $y = 4.32(1.2)^x$   
 D)  $y = 3(2)^x$



4. The equation of the graph below passing through  $(1, 1)$  is

- A)  $y = \ln x - 1$   
 B)  $y = \ln x + 1$   
 C)  $y = \ln(x - 1)$   
 D)  $y = \ln(x + 1)$



5. The value  $\log_8 0.989$  is approximately  
 A) -0.0053                      B) -0.0048                      C) -187.9985                      D) 8.0890
6. What is the future value of an annuity after investing periodic payments of \$1900 for 14 years at an annual interest rate of 5.5% compounded bi-annually?  
 A) \$78 585                      B) \$94 303                      C) \$141 454                      D) \$47 151
7. Rewrite  $\log_6 xy$  as a sum or difference or multiple of logarithms:  
 A)  $\log_6 x + \log_6 y$                       B)  $\log_6 x - \log_6 y$                       C)  $\log_3 x + \log_3 y$                       D)  $6(\log x + \log y)$
8. Suppose the algae growth in Black Oak Lake increases from 100 cells per milliliter to  $10^8$  cells per milliliter in a 4 day period. The specific growth rate  $r$  is given by the formula

$$r = \frac{\log N_2 - \log N_1}{x_2 - x_1},$$

where  $N_1$  is the algae concentration at time  $x_1$  and  $N_2$  is the algae concentration at time  $x_2$ . What is the specific growth rate of algae in Black Oak Lake?

- A) -1.50                      B) 1.50                      C) -25 000 000                      D) 25 000 000
9. Wind speed varies in the first twenty meters above the ground. For a particular day, the wind speed  $w(x)$  is measured according to  $w(x) = 1.31 \ln x + 4.1$ , where  $x$  is the height in meters above the ground. What is the wind speed 8 meters above ground?  
 A) 6.80 m/s                      B) 5.93 m/s                      C) -1.40 m/s                      D) 6.18 m/s
10. What is the annual percentage rate of interest if \$1080 grows to \$1693.78 after 9 years, where the interest is compounded continuously?  
 A) 7.2%                      B) 5%                      C) 0.05%                      D) .72%
11. How many times more intense is an earthquake measuring 8 on the Richter scale than one measuring 5?  
 A) 3 times                      B)  $10^8 - 10^5$  times                      C)  $10^3$  times                      D)  $\frac{8}{5}$  times

12. The function that represents a horizontal reflection of  $y = 2^x$  is

A)  $y = \left(\frac{1}{2}\right)^x$

B)  $y = \left(\frac{1}{2}\right)^{-x}$

C)  $y = \frac{1}{x^2}$

D)  $y = -2^x$

13. If  $f(a+x) = f(a) \cdot f(x)$ , then  $f(x)$  could be

A)  $e^x$

B)  $x^2$

C)  $\log x$

D)  $x + 3$

14. If  $f(ax) = f(a) + f(x)$ , then  $f(x)$  could be

A)  $e^x$

B)  $x^2$

C)  $\log x$

D)  $x + 3$

15. What is the pH of a solution whose hydrogen ion concentration is  $10^{-1}$  mol/L?

A) 0.1

B) 1

C) 7

D) 10

PART B - FREE RESPONSE Answer in the space provided. Show all work only for #18.

16. How long will it take to double an investment with 8% interest rate compounded quarterly?

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17. Sara repays a loan to the bank monthly over 5 years at an annual interest rate of 6%. Her monthly payment is \$550.
- How much money, to the nearest dollar, did the bank loan Sara?
  - What total amount did Sara repay to the bank?
  - How much money could Sara have earned by investing the difference in what she borrowed from what she repaid the bank for 5 years, at the same interest rate compounded continuously?

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18. As a crime scene investigator, you are required to show all your work in solving the following case. You come across a dead body in the trunk of a car. You immediately take the body's temperature and find that it is initially  $28^{\circ}\text{C}$ . The temperature in the trunk is a steady  $20^{\circ}\text{C}$ . Four hours later, you take the body's temperature again and find that it is  $24^{\circ}\text{C}$ . Knowing, as you do, that human body has a normal, living body temperature of  $37^{\circ}\text{C}$ , you can determine the time of death. How long ago did the victim die? (Round ONLY your final answer.)

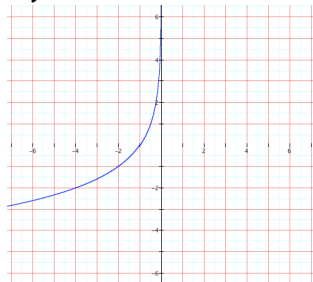
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## NO CALCULATOR SECTION

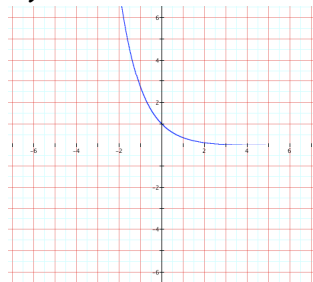
PART C - MULTIPLE CHOICE

19. The graph of  $f(x) = \log_{\left(\frac{1}{2}\right)} x$  is

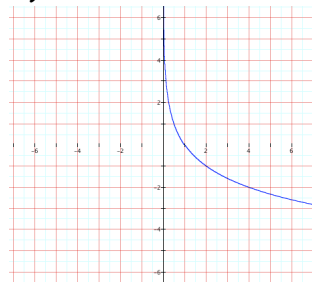
A)



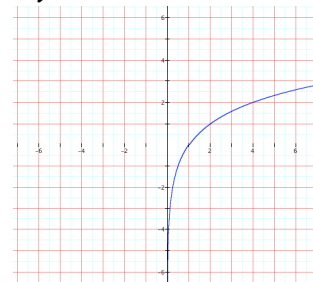
B)



C)



D)



20. Solve for  $x$ :  $\log(x+4) - \log(x-4) = \log 5$

A) -6

B) 0

C)  $\frac{5}{2}$ 

D) 6

21. The type of function and the end behaviour limits of  $f(x) = 7^{-x}$  are

A) Exponential growth;  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$

B) Exponential growth;  $\lim_{x \rightarrow -\infty} f(x) = \infty$  and  $\lim_{x \rightarrow \infty} f(x) = 0$

C) Exponential decay;  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$

D) Exponential decay;  $\lim_{x \rightarrow -\infty} f(x) = \infty$  and  $\lim_{x \rightarrow \infty} f(x) = 0$

22. Describe how to transform  $g(x) = \ln x$  to get  $f(x) = \ln(-x) + 5$ .

A) Reflect across the  $x$ -axis and then translate 5 units up.

B) Reflect across the  $y$ -axis and then translate 5 units up.

C) Translate 5 units up and then reflect across the  $x$ -axis.

D) Translate 5 units up and then reflect across the  $y$ -axis.

23. Solve for  $x$ :  $\log(3x) = \log 2 + \log(x+5)$ .

A)  $\frac{7}{2}$ 

B) 10

C) -10

D) 2

24. Which of the following is NOT equivalent to  $\log_3(4^{-1})$ ?

A)  $\frac{\ln 4^{-1}}{\ln 3}$

B)  $-\log_3 4$

C)  $-1$

D)  $\log_3\left(\frac{1}{4}\right)$

25. The two logarithms  $\log x^2$  and  $2\log|x|$  are equivalent

A)  $\forall x \in \mathbb{R}, x > 0$

B)  $\forall x \in \mathbb{R}, x \geq 0$

C)  $\forall x \in \mathbb{R}$

D)  $\forall x \in \mathbb{R}, x \neq 0$

PART D - SOLVE: Solve each equation for  $x$  algebraically. Where necessary use the natural logarithm instead of any other base logarithm. (Don't forget to check the validity of your solution.)

26.a)  $e^{2x+1} = 5$

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b)  $2(4^x) = 9(2^x) - 4$

/5

27.a)  $\log_{\sqrt{2}} x^3 = -6$

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b)  $\log(x+2) + \log(x-1) = 1$

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PART E - FREE RESPONSE

28. How do you transform  $f(x) = \log x$  to get  $y = \log_2 x$ ?

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29. How do you transform  $f(x) = e^x$  to get  $y = 3^x$ ?

/2

30. Determine the domain of  $y = \sqrt{e^x} \cdot \ln(x-1)$ .

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31. Determine the value(s) of  $k$  such that  $4^{x^2} = 2^{kx-2}$  has

a) one real solution

b) no real solutions

c) two real solutions

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32. Determine the value of  $a$  such that  $g(x) = \log_a x$  and  $g^{-1}(7) = 3$ .

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BONUS: Answer the following on the back of this sheet.

Compound Interest

a) State the limit definition of  $e$ .

b) Derive the formula for continuous compounding using the limit definition of  $e$  and the compound interest formula as the number of compoundings increases  $k \rightarrow \infty$ .

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