

PART A - CALCULATOR ACTIVE - Answer in the space provided. (Total: 22 marks)

Time: 30 minutes

1. Given the following data for Georgia's population (in millions of people)

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Pop	2.2	2.6	2.9	2.9	3.1	3.4	3.9	4.6	5.5	6.5	8.2

a) determine the exponential regression equation for Georgia's population, using 0 years for 1900 and increasing by 10.

/ 2

b) predict the population in 2005.

2. Approximate to 3 decimal places: (a) $\sqrt[3]{e} \doteq$ (b) $\log_3 11 \doteq$

/ 2

3. Which of the following is equivalent to $\log_3 11$? (Circle the correct answer.)

A) $\log_{11} 3$ B) $\frac{1}{\log_{11} 3}$ C) $-\log_{11} 3$ D) $\log_{\left(\frac{3}{11}\right)} \frac{11}{3}$ E) none of these

/ 1

4. Which of the following is equivalent to $\ln 2x$? (Circle the correct answer.)

A) $2 \ln x$ B) $\ln 2 + \ln x$ C) $(\ln 2)(\ln x)$ D) $\ln x^2$ E) none of these

/ 1

5. Which of the following is equivalent to $\ln a^3$? (Circle the correct answer.)

A) $a \ln 3$ B) $\frac{3 \log a}{\log e}$ C) $3 \ln \sqrt[3]{a}$ D) $\log_a 3$ E) none of these

/ 1

6. Consider the basic function $y = 2^x$.

a) Rewrite 2 as a power of e (i.e., determine k such that $2 = e^k$).

/ 3

b) Rewrite the function $y = 2^x$ using base e . What kind of transformation is this on $y = 2^x$?

7. A drug is administered intravenously for pain. The function $f(t) = 90 - 52\ln(1+t)$, where $0 \leq t \leq 4$, gives the amount of the drug in the body after t hours. (Express all answers to 3 decimal places.)

a) What was the initial number of units of drug administered?

/3

b) How much is present after 2 hours?

c) When will there be 50 units of the drug in the body?

8. The population of the town of Preston is currently 89 000 and is decreasing by 1.8% each year.

a) Write a function that models this population as a function of time.

/3

b) Predict when the population will be 50 000.

9. There is initially 56 g of a radioactive substance whose half-life is 18 days.

a) Write a function that models mass of the substance as a function of time.

/6

b) What is the mass of the substance after 29 days?

c) When will the substance be reduced by 75%?

PART B - NO CALCULATOR (Answer in the space provided)

(Total: 39 marks)

Time: 50 minutes10. Rewrite in (a) logarithmic form: $4^3 = 64 \Rightarrow$ (b) exponential form: $\log_x b = p \Rightarrow$

/2

11. Evaluate the following:

a) $\log_{\sqrt{2}} 1 =$

c) $\log_{\frac{1}{2}} 32 =$

e) $\log_2 0 =$

g) $6^{\log_6 13} =$

/8

b) $\log_2 256 =$

d) $\log_2 \sqrt[3]{4} =$

f) $\log_2(-8) =$

h) $\ln e^\pi =$

12. Evaluate $\log_{\frac{3}{2}} \sqrt[6]{\frac{32}{243}}$.

/3

13. Between which two integers does $\log_4 30$ lie?

/1

14. Solve for x . If it is necessary to use logarithms, use only natural logarithms.

a) $3^x = \frac{1}{9}$

c) $3^x = 7$

/10

b) $9^{2x-5} = \sqrt[4]{27}$

d) $-2(3^x)^4 + 6 = -4$

15. State the domain of $y = \ln(x-3)$.

/2

16. The logistic model is $P(t) = \frac{c}{1 + ae^{-bt}}$, where a , b and c are positive constants.

a) Determine $\lim_{t \rightarrow \infty} P(t) =$

3 b) What is the meaning of this limit to the graph of $P(t)$?

c) Why is the logistic function a more realistic model than the exponential model $P(t) = ab^t$ for population growth for large values of time, t ?

17. TRUE or FALSE: Write the full word in the blank.

10 a) _____ $\ln(x+2) = \ln x + \ln 2$	f) _____ $\log_3(7x) = 7 \log_3 x$
b) _____ $\log_2(5x) = \log_2 5 + \log_2 x$	g) _____ $\ln \frac{x}{5} = \ln x - \ln 5$
c) _____ $\log \frac{x}{4} = \frac{\log x}{\log 4}$	h) _____ $\log_4 x^3 = 3 \log_4 x$
d) _____ $\log_5 x^2 = (\log_5 x)(\log_5 x)$	i) _____ $e^{\ln 6 + \ln x} = 6x$
e) _____ $2^{x+3} = 8 \cdot 2^x$	j) _____ $\log_a a^x = \log_b b^x$

BONUS - Answer the following on looseleaf and staple the pages to PART B.

18. Determine the inverse of the function $f(x) = 2^{x-3} + 1$ and verify that your answer is

3 correct by computing either composition, $f[f^{-1}(x)]$ or $f^{-1}[f(x)]$.

19. Determine the coordinates of the point(s) of intersection of the graphs of $y = e^x$

3 and $y = xe^x$.