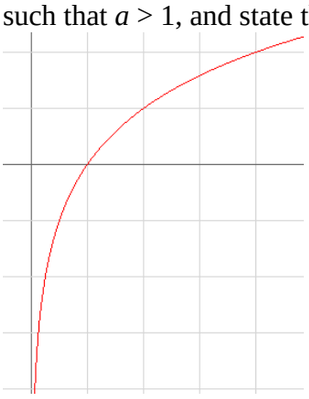
Worksheet 1: Exponential and Logarithmic Functions

1. By letting $y = x^{\frac{1}{3}}$, or otherwise, find the values of x for which $x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} = 1$.

2. Solve the equation $x^{\frac{1}{2}} + x^{-\frac{1}{2}} = 2\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$.

3. The diagram below shows the graph of $y = \ln x$. Sketch the graph of $y = \ln(x + a)$, where a is a constant such that $a > 1$, and state the coordinates of the points of intersection of the graph with the axes.



4. Given the simultaneous equations $2^x = 3^y$, $x + y = 1$, show that $x = \dfrac{\ln 3}{\ln 6}$.

5. Solve exactly $4x^{\frac{1}{2}} = 3 - 7x$.

6. By means of the substitution $y = 8^x$, or otherwise, find the exact values of x which satisfy the equation $64^x - 5(8^x) + 4 = 0$.

7. It is given that $\ln a = x$ and $\ln b = y$. Express $\ln\left(\dfrac{a^2 b}{e}\right)$ in terms of x and y.

8. The generation time for bacteria is the time that it takes for the population to double. The generation time G can be found using experimental data and the formula $G = \dfrac{t}{3.3\log_b f}$, where t is the time period, b is the number of bacteria at the beginning of the experiment, and f is the number of bacteria at the end of the experiment. The generation time for mycobacterium tuberculosis is 16 hours. How long will it take four of these bacteria to multiply into 1024 bacteria?

9. Bakersfield, California was founded in 1859 when Colonel Thomas Baker planted ten acres of alfalfa for travellers going from Visalia to Los Angeles to feed their animals. The city's population can be modelled by the equation $y = 33{,}430e^{0.0397t}$, where t is the number of years since 1950.

   a. Has Bakersfield experienced growth or decline in population? Explain.

   b. What was Bakersfield's population in 1950?

   c. Find the projected population of Bakersfield in 2010.

10. Given $\log_{10} 3 = m$, $\log_{10} e = n$, express in terms of m and/or n

   (i) $\log_{10}\left(\dfrac{3}{10}\right)$ [3 marks]

   (ii) $\log_e 9$. [4 marks]

11. The value, \$V, of a particular car can be modelled by the equation $V = ke^{-\lambda t}$ where t years is the age of the car. The car's original price was \$7499, and after one year it is valued at \$6000. State the value of k and calculate λ giving your answer to 2 decimal places. Hence obtain the value of the car when it is three years old.