2.5 Exercises

A

1. Evaluate each expression by first using the laws of logarithms.

(a) $\log_2 320 - \log_2 20$ **(b)** $\log_2 144 - \log_2 9$

(c) $\log_6 4 + \log_6 9$ **(d)** $\log 4 + \log 25$

(e) $\log_8 16 + \log_8 32$ **(f)** $\log_3 27 + \log_3 9$

2. Use the laws of logarithms to expand each expression.

(a) $\log_2 (14 \times 9)$ **(b)** $\log_5 \left(\frac{735}{40}\right)$

(c) $\log_7 (25)^{\frac{1}{2}}$ **(d)** $\log_6 (9 \times 8 \times 7)$

(e) $\log_3 (15)^4$ **(f)** $\log_4 \left(\frac{81}{30}\right)$

3. Evaluate each expression without using a calculator.

(a) $\log 25 + \log 4$ **(b)** $\log_3 18 - \log_3 6$

(c) $\log_2 8^3$ **(d)** $\log_3 \sqrt{9}$

(e) $\log_6 3 + \log_6 12$ **(f)** $2 \log_5 15 - \log_5 9$

(g) $\log_4 32 - \log_4 2$ **(h)** $\log_2 (32)^4$

4. Evaluate, using the law for logarithms of powers.

(a) $\log_3 \sqrt[3]{9}$ **(b)** $\log_3 \sqrt[4]{27}$

(c) $\log_6 \sqrt[3]{36}$ **(d)** $\log_5 \sqrt{125}$

(e) $\log_8 \sqrt[3]{64}$ **(f)** $\log_4 \left(\frac{1}{16}\right)$

5. **Knowledge and Understanding:** Evaluate and then state the logarithmic law that you used.

(a) $\log_8 6 - \log_8 3 + \log_8 2$ **(b)** $\log_2 \sqrt[3]{32}$

(c) $\log_3 54 + \log_3 \left(\frac{3}{2}\right)$ **(d)** $\log_5 \sqrt[5]{125}$

(e) $\log_8 2 + 3 \log_8 2 + \frac{1}{2} \log_8 16$ **(f)** $\log_2 \left(\frac{1}{8}\right)$

6. Express each expression as a single logarithm.

(a) $3 \log_5 2 + \log_5 7$ **(b)** $2 \log_3 8 - 5 \log_3 2$

(c) $2 \log_2 3 + \log_2 5$ **(d)** $\log_3 12 + \log_3 2 - \log_3 6$

(e) $\log_4 3 + \frac{1}{2} \log_4 8 - \log_4 2$ **(f)** $2 \log 8 + \log 9 - \log 36$

B

7. Evaluate $\log_2 (8)(32) + \log_7 (49)(\sqrt[4]{7})$.

8. Given $x = \log_2 5$ and $y = \log_2 3$, evaluate each expression in terms of x and y.

(a) $\log_2 15$ **(b)** $\log_2 0.6$ **(c)** $\log_2 125$

9. Solve for x.

(a) $\log_2 x = \log_2 5 + \log_2 10$

(b) $\log_3 x = \log_3 18 - \log_3 3$

(c) $\log x = \log 84 + \log 5 - \log 7$

(d) $\log x = 2 \log 4 + 3 \log 3$

(e) $\log_5 x - \log_5 8 = \log_5 6 + 3 \log_5 2$

10. Express as a single logarithm. Assume all variables are positive.

(a) $\log_2 x + \log_2 y + \log_2 z$ **(b)** $\log_5 u - \log_5 v + \log_5 w$

(c) $\log_6 a - (\log_6 b + \log_6 c)$ **(d)** $\log_2 x^2 - \log_2 xy + \log_2 y^2$

(e) $1 + \log_3 x^2$ **(f)** $3 \log_4 x + 2 \log_4 x - \log_4 y$

11. If $\log_3 x = 0.2$, find the value of $\log_3 x\sqrt{x}$.

12. If $\log_a w = \frac{1}{2} \log_a x + \log_a y$, express w in terms of x and y.

13. **Communication:** Explain the similarities between the laws of exponents and the laws of logarithms.

14. Use a calculator to evaluate each expression to two decimal places.

(a) $\log 4^8$ **(b)** $\log \sqrt{40}$ **(c)** $\log 9^4$

(d) $\log 200 \div \log 50$ **(e)** $(\log 20)^2$ **(f)** $5 \log 5$

15. **Application:** The loudness, L, of a sound is related to the sound's intensity, I, by $L = 10 \log \frac{I}{I_0}$, where L is measured in decibels, I is measured in watts per square metre, and I_0 is the intensity of a barely audible sound. By how many decibels does the loudness increase if the intensity of the sound from a tuning fork is tripled?

16. A barely audible sound has an intensity of $I_0 = 10^{-12}$ W/m^2.
Use the formula in question 15 to calculate the loudness of each sound.

(a) a falling pin: $I = 10^{-11}$ W/m^2

(b) quiet conversation: $I = 10^{-6}$ W/m^2

(c) subway: $I = 10^{-3}$ W/m^2

(d) jet at take-off: $I = 1$ W/m^2