Chapter 4, Review and Practice, page 378

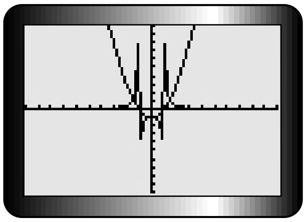
1. Vertex form is $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex. This form is found by transforming the equation in standard form by completing the square. A quadratic function may be in standard form: $f(x) = ax^2 + bx + c$, where $(0, c)$ is the y-intercept, or in factored form: $f(x) = a(x - p)(x - q)$, where $(p, 0)$ and $(q, 0)$ are the x-intercepts.
2. To complete the square: factor the coefficient of x^2 from the first two terms; add and subtract the square of half the coefficient of x inside the brackets; group the three terms that form the perfect square; multiply the fourth term by a, and move it outside the brackets; factor the perfect square and simplify.
3. **(a)** $f(x) = (x - 3)^2 - 2$ **(b)** $f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{21}{4}$
   (c) $f(x) = 2\left(x + \frac{7}{4}\right)^2 - \frac{73}{8}$ **(d)** $f(x) = -4\left(x + \frac{3}{8}\right)^2 + \frac{41}{16}$
   (e) $f(x) = -1.6(x + 0.75)^2 - 0.1$
   (f) $f(x) = \frac{2}{3}\left(x - \frac{1}{3}\right)^2 + \frac{52}{27}$
4. **(a)** $y = (x + 4)^2 - 3$, vertex: $(-4, -3)$, D = **R**, R = $\{y \mid y \geq 3, y \in \mathbf{R}\}$; parabola, opens up, zeros at -5.67, -2.27; y-intercept: 7
   (b) $y = -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8}$, vertex: $\left(\frac{5}{4}, \frac{25}{8}\right)$, D = **R**, R = $\{y \mid y \leq \frac{25}{8}, y \in \mathbf{R}\}$; parabola, opens down, zeros at 0, 2.5; y-intercept: 0
   (c) $y = \frac{1}{2}(x - 5)^2 + \frac{5}{2}$, vertex: $\left(5, \frac{5}{2}\right)$, D = **R**, R = $\{y \mid y \geq \frac{5}{2}, y \in \mathbf{R}\}$; parabola, opens up, no zeros; y-intercept: 15
5. 0.8 m
6. **(a)** *Example*: y-intercept: for $f(x) = 2x^2 + 6x + 1$, (0, 1) is y-intercept.
   (b) *Example*: x-intercepts: for $f(x) = (x - 5)(x - 2)$, (5, 0) and (2, 0) are x-intercepts.
   (c) *Example*: vertex: for $f(x) = (x - 2)^2 + 3$, vertex is (2, 3).
7. The graph of $g(x)$ is graph of $f(x)$, translated c units down; therefore, to find the maximum and minimum points of $g(x)$, one can just add c to the maximum and minimum points of $f(x)$. To find the maximum or minimum points of $f(x)$, factor: $ax^2 + bx = x(ax + b)$. Therefore, the intercepts are (0, 0) and $\left(-\frac{b}{a}, 0\right)$. The maximum and minimum points are halfway between these points, so $-\frac{b}{2a}$ is the x-coordinate of the vertex for both functions. To find the y-coordinate of the vertex for $g(x)$, substitute $x = -\frac{b}{2a}$ into the equation. This y-value is the maximum or minimum value of $g(x)$. To find the maximum or minimum value for $f(x)$, subtract c from the y-coordinate for the vertex for $g(x)$.
8. When the coefficient of the x^2 term is positive, the function has a minimum value; when negative, it has a maximum value.
   (a) maximum, $x = 1$, 6 **(b)** minimum, $x = -2$, -21
   (c) maximum, $t = 2$, 0 **(d)** minimum, $x = 0.55$, -0.672
   (e) minimum, $x = 1.1$, -4.5
   (f) minimum, $x = -225$, -1827
9. **(a)** Revenue $= -5x^2 + 22x$ **(b)** $P(x) = -5x^2 + 19x - 15$
   (c) $x = 1.9$ **(d)** 1100 or 2700
   (e) $P(x) = -5(x - 1.9)^2 + 3.05$; parabola, opens down, vertex (1.9, 3.05) zeros at 1.12 and 2.68; y-intercept: -15
10. 8 empty seats
11. *Examples*:
    parabola opens up, with vertex above x-axis or parabola opens down, with vertex below x-axis; no zeros since graph does not cross x-axis.
    (a) parabola opens up or down, vertex on x-axis; one zero since graph touches x-axis once.
    (b) parabola opens up, vertex below x-axis or parabola opens down, vertex above x-axis; two zeros since graph crosses x-axis twice.
12. discriminant: $b^2 - 4ac$; if $b^2 - 4ac = 0$, function has 1 zero; if $b^2 - 4ac > 0$, it has 2 zeros; if $b^2 - 4ac < 0$, it has no zeros.
    Examples:
    (a) $4x^2 - 5x + 2$ **(b)** $x^2 - 2x + 1$
    (c) $8x^2 + 9x + 1$
13. When the signs of a and k are opposite, function has 2 zeros; when they are the same, it has no zeros; when $k = 0$, it has 1 zero.
    Examples:
    (a) $y = 4(x - 3)^2 + 1$ **(b)** $y = 2(x - 1)^2$
    (c) $y = -3(x - 1)^2 + 2$
14. **(a)** 2 zeros, 2 points **(b)** 2 zeros, 2 points
    (c) one zero, one point **(d)** no zeros, no points
    (e) 1 zero, 1 point **(f)** 2 zeros, 2 points
15. **(a)** $k < 8$, $k = 8$, $k > 8$
    (b) $k > 20$ or $k < -20$, $k = \pm 20$, $-20 < k < 20$
    (c) $k > 12$ or $k < -12$, $k = \pm 12$, $-12 < k < 12$
    (d) $k > 1$ or $k < -3$, $k = -3$ or $k = 1$, $-3 < k < 1$
16. **N** = set of natural numbers, **W** = set of whole numbers, **I** = set of integers, **Q** = set of rational numbers, $\overline{\mathbf{Q}}$ = set of irrational numbers, **R** = set of real numbers, and **C** = set of complex numbers.
17. Complex numbers are expressed in the form $a + bi$, where a and b are real and $i^2 = -1$.
18. The quadratic equation $ax^2 + bx + c$ has no real roots when $b^2 - 4ac = < 0$. The two complex roots are $a + bi$ and $a - bi$, which are conjugates of one another.
19. **(a) Q, R, C** **(b) I, Q, R, C** **(c) C**
    (d) $\overline{\mathbf{Q}}$**, R, C** **(e) C** **(f)** $\overline{\mathbf{Q}}$**, R, C**
    (g) $\overline{\mathbf{Q}}$**, R, C** **(h)** $\overline{\mathbf{Q}}$**, R, C** **(i) W, R, C**
    (j) N, W, I, Q, R, C
20. **(a)** $2 + i$ **(b)** $1 - 3i$ **(c)** $-4 + 5i$
    (d) $-5 - 6i$ **(e)** $13 + 13i$ **(f)** $3 - \sqrt{3}i$
21. **(a)** $\pm 9i$ **(b)** $\pm 6i$ **(c)** $1 \pm \sqrt{2}i$
    (d) $2 \pm 4i$ **(e)** $\frac{3 \pm \sqrt{7}i}{4}$ **(f)** $\frac{-6 \pm \sqrt{6}i}{3}$
22. The sum or difference of complex numbers consists of the sum or difference of the real numbers and the sum or difference of the imaginary numbers. For example, $(4 + 6i) + (5 - 2i) = 9 + 4i$. The sum of complex conjugates is a real number, with no imaginary part. For example, $(4 + 6i) + (4 - 6i) = 8$.
23. Use the distributive law to multiply complex numbers. For example, $(4 + 6i)(2 - i) = 8 - 4i + 12i - 6i^2 = 14 + 8i$. The product of two complex conjugates is a real number, with no imaginary part. For example, $(2 - i)(2 + i) = 5$.
24. **(a)** -9 **(b)** i **(c)** $12 - i$
    (d) $-18 + i$ **(e)** 50 **(f)** $5 - 3i$
    (g) $-15 - 8i$ **(h)** $\frac{1}{5}$ **(i)** $58 - 14i$
    (j) $7 + 4i$
25. **(a)** 10 **(b)** 25 **(c)** 29
26. $-11 + 26i$
27. **(a)** $-3i$, $x^2 + 9 = 0$ **(b)** $1 + 2i$, $x^2 - 2x + 5 = 0$
    (c) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $x^2 + x + 1$
28. It displays inverse variation because y decreases as x increases. Value of xy remains constant; reciprocal function graph; D = $\{x \mid x \neq 0, x \in \mathbf{R}\}$, R = $\{y \mid y \neq 0, y \in \mathbf{R}\}$.

29. The x-intercept(s) of $f(x)$ will give the equation(s) of the vertical asymptotes for $y = \frac{1}{f(x)}$. The y-intercept of $y = \frac{1}{f(x)}$ is $y = \frac{1}{y\text{-intercept of } f(x)}$. Find the points $(x, 1)$ and $(x, -1)$ of $f(x)$: graph of $y = \frac{1}{f(x)}$ will also go through these points.

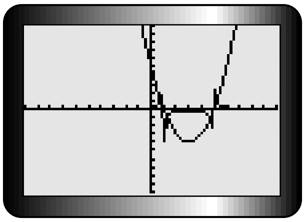
30. **(a)** **i.** inverse variation **ii.** inverse variation **iii.** no inverse variation
(b) **i.** $y = \frac{60}{x}$ **ii.** $y = \frac{2880}{x}$ **iii.** $y = 2x^2 - 1$

31. 4 s, $a = \frac{60}{t}$, hyperbola in first and third quadrants; vertical asymptote $x = 0$; horizontal asymptote $y = 0$

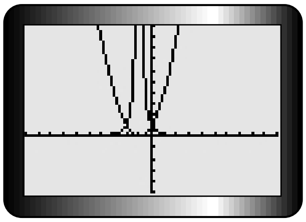
32. **(a)**



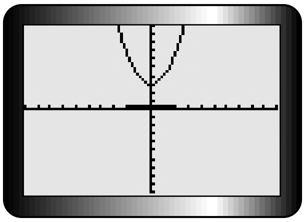
(b)



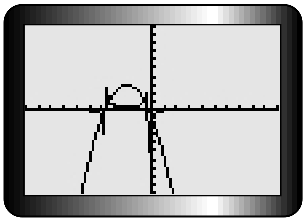
(c)



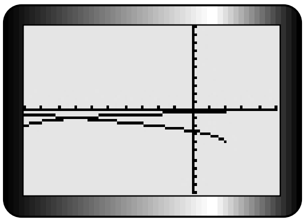
(d)



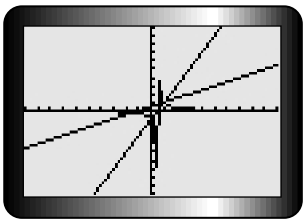
(e)



(f)

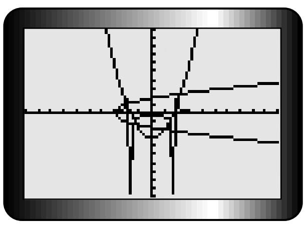


33. **(a)**



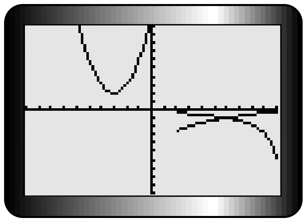
$f(x)$: D = $\mathbf{R}$, R = $\mathbf{R}$; $\frac{1}{f(x)}$: D = $\{x \mid x \neq \frac{1}{2}, x \in \mathbf{R}\}$, R = $\{y \mid y \neq 0, y \in \mathbf{R}\}$, $f^{-1}(x)$: D = $\mathbf{R}$, R = $\mathbf{R}$

(b)



$f(x)$: D = $\mathbf{R}$, R = $\mathbf{R}$; $\frac{1}{f(x)}$: D = $\{x \mid x \neq \pm 3, x \in \mathbf{R}\}$, R = $\{y \mid y \neq 0, y \in \mathbf{R}\}$, $f^{-1}(x)$: D = $\{x \mid x \geq -3, x \in \mathbf{R}\}$, R = $\mathbf{R}$

(c)



$f(x)$: D = $\{x \mid x \geq 2, x \in \mathbf{R}\}$, R = $\{y \mid y \geq -3, y \in \mathbf{R}\}$;
$\frac{1}{f(x)}$: D = $\{x \mid x \geq 2, x \in \mathbf{R}\}$, R = $\{y \mid y \leq -\frac{1}{3}, y \in \mathbf{R}\}$,
$f^{-1}(x)$: D = $\mathbf{R}$, R = $\{y \mid y \geq 2, y \in \mathbf{R}\}$

34. sum, difference, product or quotient of two polynomial functions; to find common factors.

35. when the denominator is 0; must state restrictions on the variables

36. **(a)** $-\frac{3}{5}, x \neq -3$ **(b)** $-\frac{8p^2}{3q}, p, q \neq 0$
(c) $\frac{4x+3}{x+1}, x \neq -1, \frac{3}{4}$ **(d)** $\frac{a+2}{a+3}, a \neq -3, 4$
(e) $\frac{2x-3}{2x+3}, x \neq -\frac{2}{5}, -\frac{3}{2}$ **(f)** $\frac{x-3}{-(3+x)}, x \neq 3, -3$
(g) $\frac{x-2y}{-(y-x)}, x \neq \frac{y}{2}, y$

37. **(a)** 1.1, 1.7, 1.7 **(b)** $4.5h$

38. factor numerator and denominator, state restrictions, divide out common factors, multiply the numerators and multiply the denominators, then express as a single rational expression.

39. the reciprocal is used

40. **(a)** $\frac{y}{2}, x, y \neq 0$ **(b)** $m, m, n \neq 0$
(c) $\frac{2b}{3c^2}, a, b, c \neq 0$ **(d)** $\frac{5}{2p}, p, q \neq 0$

41. **(a)** $\frac{1}{(x+3)(x-2)}, x \neq 2, \pm 3$ **(b)** $\frac{2}{-a^2}, a \neq 0, 5$
(c) $\frac{1}{2(y-5)}, y \neq \pm 3, 5$
(d) $\frac{(x-3)^2(x-5)}{(x+2)^2(x-8)}, x \neq -7, -2, 8$

42. **(a)** -4 **(b)** 2 **(c)** $\frac{1}{5}$
(d) $\frac{5}{4}$ **(e)** undefined **(f)** 0
(g) $-\frac{2(x+3)}{(x-3)}, x \neq 0, \pm 3$
$f(x)$: $x \neq 0, 3$; $g(x)$: $x \neq 0, 2$

43. **(a)** $\frac{x}{36y}, x \neq 0, y \neq 0$
(b) $\frac{(x-3)(x+1)}{(x-1)}, x \neq \pm 1, \pm 2, 5$
(c) $\frac{x(1-x)}{-y(y+1)}, x \neq 0, -1, y \neq 0, \pm 1$
(d) $(x-y), y \neq \pm 2x, -x, -\frac{2}{3}x$
(e) $\frac{a+b}{a-b}, a \neq \pm 3b, \pm 2b, \pm b$

44. product is a real number

45. multiply numerator and denominator by the complex conjugate of the denominator

46. **(a)** $2 + i, \frac{1}{2-i}, \frac{2}{5} + \frac{1}{5}i$ **(b)** $-1 - i, \frac{1}{-1+i}, -\frac{1}{2} - \frac{1}{2}i$
(c) $\sqrt{2} + \sqrt{2}i, \frac{1}{\sqrt{2} - \sqrt{2}i}, \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}i$
(d) $-3 - \sqrt{3}i, \frac{1}{-3+\sqrt{3}i}, -\frac{1}{4} - \frac{1}{4\sqrt{3}}i$
(e) $-\frac{1}{2} - \frac{\sqrt{3}}{2}, \frac{2}{-1+\sqrt{3}i}, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

47. **(a)** $2 + 2i$ **(b)** $2 - i$ **(c)** $3 + 3i$
(d) $\frac{4}{5} + \frac{12}{5}i$ **(e)** $1 - i$ **(f)** $\frac{11}{13} + \frac{16}{13}i$
(g) $-1 - 5i$ **(h)** $-4 - 3i$
(i) $\frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2}i$ **(j)** $2 - 3i$

(k) $-\frac{7}{25} + \frac{74}{25}i$

48. **(a)** $\frac{13\sqrt{3}}{4} - \frac{5}{4}i$

(b) real: $\frac{x^3 + xy^2 + x}{x^2 + y^2}$, imaginary: $\frac{x^2y + y^3 - y}{x^2 + y^2}$

49. **(a)** $\frac{1}{2} - \frac{7}{2}i$ **(b)** $\frac{2}{5} + \frac{14}{5}i$

50. must have common factors;

Example: $\frac{9}{x^2 + x - 12} + \frac{5}{x^2 - 9} = \frac{14x + 47}{(x + 4)(x - 3)(x + 3)}$

51. must include all points where the function is undefined

52. **(a)** $\frac{2}{15x}, x \neq 0$ **(b)** $\frac{3a + 5b}{a^2b^2}, a, b \neq 0$

(c) $\frac{3x - 7}{(x + 1)(x - 1)}, x \neq 1$ **(d)** $\frac{x - 9}{(x + 3)^2}, x \neq -3$

(e) $\frac{2(x - 2)}{(x + 4)(x - 1)(x - 3)}, x \neq -4, 1\ 3$

(f) $\frac{5}{(x - 2)(x - 3)(x + 3)}, x \neq \pm 3, 2$

(g) $\frac{x}{2y}, x \neq 0, -2y, y \neq 0$

(h) $\frac{6}{(x + 3)(x - 1)(x + 1)}, x \neq \pm 1, -3$

(i) $\frac{3}{(x + 1)(x - 2)}, x \neq -1, 2, -3$

(j) $\frac{-6b}{(a + b)(a - b)}, a \neq 0, b, b \neq 0$

53. **(a)** $x = \frac{-3}{2}, \frac{5}{2}$ **(b)** $x = \frac{-5}{3}, \frac{1}{3}$

54. **(a)** $x = 4, 7$ **(b)** $x = \frac{-1 + \sqrt{7}}{3}, \frac{-1 - \sqrt{7}}{3}$

(c) $x = \pm\frac{1}{3}, 0$ **(d)** $x = \frac{5}{3}$

55. **(a)** Tom: 20.0 m/s, Genna: 25.0 m/s

(b) Tom: 32.5 s, Genna 26 s

56. *Example:* $p(x) = 5x^3 + x + 1$, $q(x) = 2x^2 + x + 3$, $r(x) = x + 6$, degree = 3, 2, 1; exponents are non-negative integers; degree is the highest exponent of the variable

57. *Example:* $p(x) + q(x) = 5x^3 + 2x^2 + 2x + 4$;
$p(x) - r(x) = 5x^3 - 5$; $q(x) * r(x) = 2x^3 + 13x^2 + 9x + 18$

58. **(a)** *Example:* $(x + 1)(x^2 - x + 6) + x - (x + 2)$, degree = 3

(b) *Example:* $(x + 1)(x - 1)(2x + 2) + (x - 3) - 2$, degree = 3

(c) *Example:* $(x^2 + x + 1)^2 + x - (x + 2)$, degree = 4

59. **(a)** $4x^4 + 2x^3 - 4x^2 + 3$ **(b)** $-2x(x + 1)$

(c) $x(-4x^2 + 9x + 14)$

(d) $x(-3x^5 + 8x^4 - 5x^3 - 3x^2 + 12x)$

(e) $x(11x - 8)$ **(f)** $5x(x - 4)$

60. **(a)** $x^4 - 2x^3 + 7x^2 - 8x + 12$ **(b)** $x(25x^4 - 14x^2 + 1)$

(c) $11t^3 - 20t^2 + 3t - 14$ **(d)** $10x^2 - 20x - 55$

(e) $3x^4 + 10x^3 - 14x^2 + x + 2$

(f) $3a^4 + 10a^3 - 13a^2 + 22a - 12$

(g) $x^4 - 6x^3 + 13x^2 - 12x + 4$

(h) $x^4 - 6x^3 + x^2 + 24x + 16$