Investigating Transformations 5.6

In section 5.3, you were able to find Sarah and Billy's location anywhere on the London Eye Ferris wheel by knowing the radius of the Ferris wheel and the angle of rotation. You also found that any point on the circumference of the wheel has coordinates $(r \cos \theta, r \sin \theta)$. You saw that the functions $y = \cos \theta$ and $y = \sin \theta$ are models for the x- and y-coordinates, respectively, as the point $P(x, y)$ moves around the unit circle.

Part 1: The Graphs of $y = a \sin \theta$ and $y = a \cos \theta$

Sarah and Billy found information on the Internet about another large Ferris wheel. This Ferris wheel is in Osaka, Japan, and its diameter is 100 m. The height of the Ferris wheel from the ground to the highest point is 112.5 m.

Think, Do, Discuss

1. (a) Suppose that the centre of the wheel is at the origin of a graph. What are the coordinates of any point on the circumference of the Ferris wheel in Osaka?

(b) State the function that models the y-coordinate for any angle of rotation, θ. Graph the function for $0 \le \theta \le 4\pi$, and state its amplitude.

(c) State the function that models the θ-coordinate for any angle of rotation, θ. Graph the function for $0 \le \theta \le 4\pi$, and state its amplitude.

2. (a) Predict the graph of $y = a \sin \theta$ for $0 \le \theta \le 4\pi$, for $a = 1, 2$, and 3, and for $a = \frac{1}{2}$ and $\frac{1}{4}$. Sketch each graph on the same axes. Verify your sketches with a graphing calculator.

(b) Sketch, on a new set of axes, the graph of $y = a \sin \theta$ for $0 \le \theta \le 4\pi$, and for $a = -1, -2$, and -3. Verify your sketches with a graphing calculator. How are these graphs different from those for $a = 1, 2$, and 3?

(c) Compare the zeros for each function.

(d) How is the amplitude of each graph affected by the value of a?

(e) What are the maximum and minimum values for each graph?

3. Repeat step 2 for $y = a \cos \theta$.

4. Explain how the value of a affects $y = a \sin \theta$ and $y = a \cos \theta$.

Part 2: The Graphs of $y = a \sin \theta + d$ and $y = a \cos \theta + d$

Think, Do, Discuss

1. (a) Sketch a model of the Osaka Ferris wheel so that the θ-axis represents the ground and the centre of the wheel is on the y-axis.

(b) Describe the transformation of any point, $P(x, y)$, on the circumference of the circle if the centre of the Ferris wheel is not at the origin, but at the point in (a).

(c) State the function that models the y-coordinate for any angle of rotation, θ. Graph the function for $0 \leq \theta \leq 4\pi$ and state its amplitude.

(d) Describe the relation between the amplitude and both the maximum and minimum values.

(e) Suppose that Sarah and Billy were riding on the Ferris wheel in Osaka. What is their height above the ground when the angle, in standard position, of the Ferris wheel is $\frac{2\pi}{3}$?

(f) Sarah and Billy are 50 m above the ground. What is the angle of rotation, in standard position, at this point?

2. (a) Predict the graph of $y = \sin \theta + d$ for $d = -2, -1, 1,$ and 2. Sketch each graph on the same axes. Verify each sketch with a graphing calculator.

(b) How can you determine the maximum and minimum values without drawing the graphs?

(c) State the maximum and the minimum values for $y = 4 \sin \theta - 9$. Explain how you calculated the values.

3. (a) Sketch, on the same axes, the graph of $y = \cos \theta + d$ for $0 \leq \theta \leq 4\pi$, for $d = -2, -1, 1,$ and 2. Verify each sketch with a graphing calculator.

(b) State the maximum and minimum values for $y = -3 \cos \theta + 5$. What is the equation of the axis of the curve? How did you find this equation?

(c) Explain how the value of d affects $y = a \sin \theta + d$ and $y = a \cos \theta + d$.

Part 3: The Graphs of $y = a \sin k\theta + d$ and $y = a \cos k\theta + d$

Sarah and Billy ride on the Ferris wheel at the fall fair. The Ferris wheel is 10 m in diameter and it is 11 m at its highest point. One revolution of the wheel takes 2 min, and the ride lasts 4 min.

Think, Do, Discuss

1. (a) What is Sarah and Billy's height on this Ferris wheel at any angle of rotation, θ?

(b) Through how many degrees does the Ferris wheel rotate in the first second? How many degrees has the Ferris wheel rotated after 2 s? 3 s? 4 s? Through how many degrees has it rotated after t seconds?

(c) Write the angle of rotation, θ, in terms of time, t.

(d) Let time be the independent variable. Describe their position by writing two trigonometric equations. Use a graphing calculator to graph the equations for a complete ride. What mode should you set for the graphing calculator? Explain.

(e) What is the period of each graph? How might you determine the period from each equation?

2. Predict the period of the graph of $y = \sin k\theta$ for $k = 2$, 3, and 4. Verify each period by drawing each graph. Clear the previous equation from the graphing calculator before entering another equation.

3. Repeat step 2 for $k = \frac{1}{2}$ and $\frac{1}{4}$. Set the window on the graphing calculator so that you may see one complete cycle of each graph.

4. How could you find the period of $y = \sin k\theta$ from the period of $y = \sin \theta$?

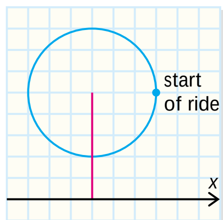
5. What is the period of $y = \cos 6\theta$? Draw the graph and visually verify the period.

6. State the period of $y = -3 \cos (2\theta) + 7$.

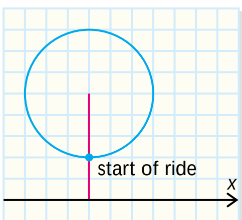
7. Explain how the value of k affects $y = a \sin k\theta + d$ and $y = a \cos k\theta + d$.

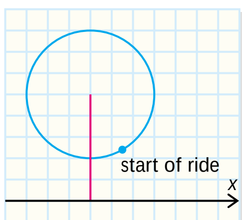
Part 4: The Graphs of $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$

Imagine that the car in which Sarah and Billy ride the Ferris wheel may hold several passengers. At the start of the ride, a number of other passengers got on with Sarah and Billy. The trigonometric functions $y = a \sin k\theta + d$ and $y = a \cos k\theta + d$ apply in this situation. This diagram represents the angle of rotation in standard position so that one arm of the angle is parallel to the x-axis.



How would the sinusoidal model change if the passengers boarded the Ferris wheel at a different point on the wheel? The diagrams show two possible points on the wheel.

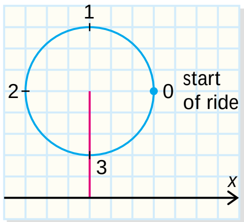




Think, Do, Discuss

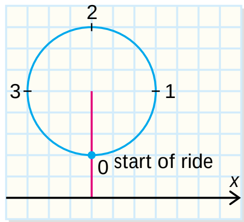
1. Sarah and Billy ride the Ferris wheel at a local fair. The lowest point on the wheel is 1 m above the ground. The highest point is 13 m above the ground. The wheel is divided into quarters labelled 0, 1, 2, and 3. Extend the pattern and complete each table. Graph each set of coordinate pairs on the same axes in the *x*-*y* plane.

(a)



Position on Wheel	0	1	2	3	4	5	6
Height (m)							

(b)



Position on Wheel	0	1	2	3	4	5	6
Height (m)							

2. (a) Highlight the first complete cycle for each curve.

(b) Find the equation of the axis for each curve. Compare the equations. What is the amplitude of each curve?

(c) Compare the maximum and minimum values of each curve.

(d) Compare the periods of the curves.

(e) How are the curves different?

(f) By how many degrees is each curve separated? Explain the difference.

(g) State the equation of the curve in step 1(a) in terms of the angle of rotation, θ. Verify that the graph of this equation matches your graph.

(h) Replace θ with $(\theta - 90°)$ in the equation. Use a graphing calculator to graph the equation and compare it to the corresponding hand-drawn graph in step 1(b).

(i) Describe the transformation of the original graph in step 1(a) to the graph in step 1(b). Explain the change in terms of the starting position.

3. Suppose the function for the height, h, in metres on the Ferris wheel is modelled by $h(\theta) = 6 \sin(\theta - 45°) + 7$.

(a) Sketch the starting point on the Ferris wheel.

(b) Graph the equation by hand.

(c) Verify your hand-drawn graph using a graphing calculator.

4. Sketch the starting point on a circle and graph each function.

(a) $h(\theta) = 6 \sin(\theta + 90°) + 7$

(b) $h(\theta) = 6 \sin(\theta + 45°) + 7$

5. (a) The Ferris wheel's speed in step 1(a) is one revolution every 1.5 min. Write the equation to represent the height if the angle at the boarding point is in the standard position. Graph the equation.

(b) Replace θ with $(\theta - 45°)$ in the equation. Graph the new equation. Describe the horizontal shift of the graph of the equation with θ to the graph of the equation with $(\theta - 45°)$. What is the starting position on the Ferris wheel? Describe this point in terms of the angle in standard position.

6. State the horizontal shift for $y = 2 \sin 3(\theta + 90°) + 5$ and compare it to the horizontal shift for $y = 2 \sin (3\theta + 90°) + 5$.

7. Explain how the value of b affects $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$.

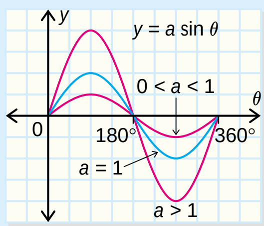
Focus 5.6

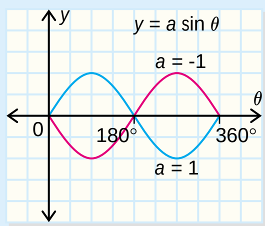
Key Ideas

- The graph of $y = a \sin k(\theta + b) + d$ is a transformation of the graph of $y = \sin \theta$. The graph of $y = a \cos k(\theta + b) + d$ is a transformation of the graph of $y = \cos \theta$. The values of a, k , b, and d determine the shape or positioning of the graph. In the following, "the graph" refers to either the graph of $y = \sin \theta$ or the graph of $y = \cos \theta$.

  The value of a determines the vertical stretch or compression, called the **amplitude**. The value of a also tells whether the curve is reflected in the θ-axis.

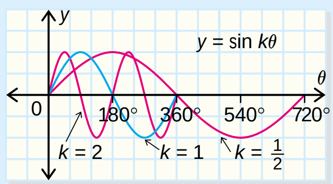
  - For $a > 1$, the graph is stretched vertically by the factor a. For $0 < a < 1$, the graph is vertically compressed by a factor of a.
  - For $a < 0$, the graph is reflected in the θ-axis and stretched vertically or compressed by the factor a.





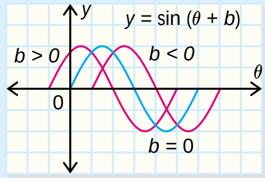
  The value of k determines the horizontal stretch or compression.

  - For $k > 1$, the graph is compressed horizontally by the factor $\frac{1}{k}$. For $0 < k < 1$, the graph is stretched horizontally by the factor $\frac{1}{k}$.
  - The value of k determines the number of cycles in the period of the graph.



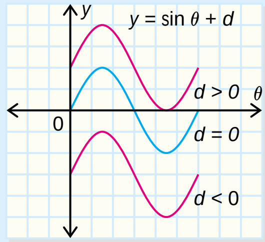
The value of b determines the horizontal translation.

- For $b > 0$, the graph is horizontally translated b units to the left.
- For $b < 0$, the graph is horizontally translated b units to the right.
- The value b is called the **phase shift**.



The value of d determines the vertical translation.

- For $d > 0$, the graph is vertically translated d units up.
- For $d < 0$, the graph is vertically translated d units down.



- To determine the phase shift of $y = \sin(k\theta + p)$ or $y = \cos(k\theta + p)$, rewrite the equations in the form $y = \sin k(\theta + b)$ or $y = \cos k(\theta + b)$, where $b = \frac{p}{k}$.
- To determine the period of the trigonometric function, divide the period of the base curve by k.

$$y = \sin k\theta \text{ has period } \frac{360°}{k} \text{ or } \frac{2\pi}{k}$$

$$y = \cos k\theta \text{ has period } \frac{360°}{k} \text{ or } \frac{2\pi}{k}$$

$$y = \tan k\theta \text{ has period } \frac{180°}{k} \text{ or } \frac{\pi}{k}$$

- To graph $y = a \sin k(\theta + b) + d$ and $y = a \cos k(\theta + b) + d$, apply the transformations in this order:

1. horizontal stretch or compression
2. horizontal translation or phase shift, left or right
3. vertical stretch or compression
4. reflection about the θ-axis
5. vertical translation, up or down

Example 1

(a) Determine the period, in degrees, of $y = \sin 3\theta$.

(b) Find the period, in radians, for $y = \cos \frac{\theta}{4}$.

Solution

(a) The period of $y = \sin \theta$ is 360°. The graph of $y = \sin 3\theta$ is compressed horizontally by a factor of $\frac{1}{3}$. There are three cycles of $y = \sin 3\theta$ for one cycle of $y = \sin \theta$. Then $3\theta = 360°$.

$$3\theta = 360°$$
$$\theta = \frac{360°}{3}$$
$$\theta = 120°$$

The period of $y = \sin 3\theta$ is 120°.

(b) The period of $y = \cos \theta$ is 2π. The graph of $y = \cos \frac{\theta}{4}$ is stretched horizontally by a factor of $\frac{1}{\frac{1}{4}}$ or 4. There is $\frac{1}{4}$ cycle of $y = \cos \frac{\theta}{4}$ for one cycle of $y = \cos \theta$.

Then $\frac{\theta}{4} = 2\pi$.

$$\theta = 4(2\pi)$$
$$\theta = 8\pi$$

The period of $y = \cos \frac{\theta}{4}$ is 8π.

Example 2

State the phase shift for each function.

(a) $y = \cos (4\theta + 180°)$

(b) $y = \sin 2(\theta + 15°)$

(c) $y = \cos (3\theta - \pi)$

(d) $y = \tan (2\theta + 180°)$

Solution

(a) Rewrite $y = \cos (4\theta + 180°)$ as $y = \cos 4\left(\theta + \frac{180°}{4}\right)$.
Then $y = \cos 4(\theta + 45°)$.
The phase shift is $-45°$.

(b) $y = \sin 2(\theta + 15°)$ is already in the correct form.
The phase shift is $-15°$.

(c) $y = \cos (3\theta - \pi)$
$= \cos 3\left(\theta - \frac{\pi}{3}\right)$
The phase shift is $\frac{\pi}{3}$.

(d) Rewrite $y = \tan (2\theta + 180°)$ as $y = \tan 2(\theta + 90°)$.
The phase shift is $-90°$.

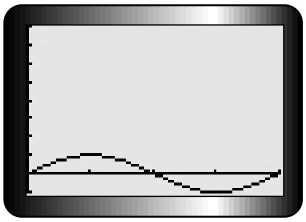
Example 3

Describe the graph of $y = -3\sin(5\theta + \pi) + 4$ for $0 \le \theta \le 2\pi$ as a transformation of $y = \sin\theta$. Graph each step of the transformation process.

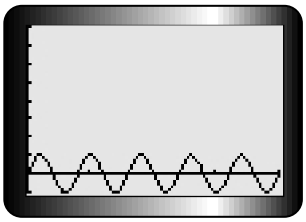
Solution

The equation is in the form $y = a\sin(k\theta + p) + d$, which gives the values of a, k, and d. Rewrite the equation in the form $y = a\sin k(\theta + b) + d$ to determine the value of b. The equation becomes $y = -3\sin 5\left(\theta + \frac{\pi}{5}\right) + 4$.

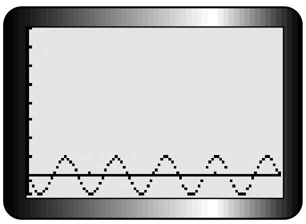
Start by graphing $y = \sin\theta$.



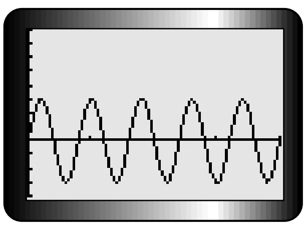
There are five cycles of the graph in the period of $y = \sin\theta$, which is 2π. The period is then $\frac{2\pi}{5}$.



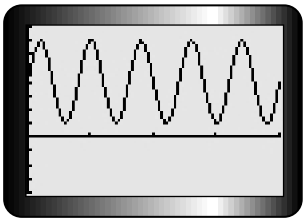
The phase shift, b, is $-\frac{\pi}{5}$. The graph shifts $\frac{\pi}{5}$ to the left.



The vertical stretch, a, is 3. Stretch the curve by a factor of 3. Because a is negative, the curve is reflected about the θ-axis.



The vertical translation, d, is 4. Translate the graph 4 units up.



Example 4

The average monthly temperature, T, in degrees Celsius in the Kawartha Lakes was modelled by $T(t) = -22 \cos \frac{\pi}{6}t + 10$, where t represents the number of months. For $t = 0$, the month is January; for $t = 1$, the month is February, and so on.

(a) What is the period? Explain the period in the context of the problem.

(b) What is the maximum temperature? the minimum temperature?

(c) What is the range of temperatures for this model?

Solution

(a) The period of $f(\theta) = \cos \theta$ is 2π.

Then
$$\frac{\pi}{6}t = 2\pi.$$
$$t = (2\pi)\left(\frac{6}{\pi}\right)$$
$$t = 12$$

The period of this function is 12.

The cycle of temperatures repeats itself every 12 months.

(b) The amplitude is $|-22| = 22$. The vertical translation is 10 units up. The maximum temperature is 22°C + 10°C = 32°C. The minimum temperature is −22°C + 10°C or −12°C.

(c) The temperature range is −12°C to 32°C.

Practise, Apply, Solve 5.6

A

1. Explain how each graph is different from the graph of $y = \sin \theta$, where $\theta \in \mathrm{R}$.

(a) $y = \sin \theta + 2$ (b) $y = \sin 2\theta$ (c) $y = 2 \sin \theta$ (d) $y = -2 \sin \theta$

2. For each function, explain the transformation from $y = \cos \theta$.

(a) $y = \frac{1}{2} \cos \theta$ (b) $y = \cos \frac{\theta}{2}$

(c) $y = \cos \theta + \frac{1}{2}$ (d) $y = -\frac{1}{2} \cos \theta$

3. State the period of each function in degrees.

(a) $y = \sin 3\theta$ (b) $y = \cos \frac{\theta}{4}$ (c) $y = \cos 6\theta$

(d) $y = \sin 5\theta$ (e) $y = \cos \frac{3}{2}\theta$ (f) $y = \sin \frac{\theta}{3}$

(g) $y = \tan 2\theta$ (h) $y = \tan \frac{\theta}{2}$ (i) $y = \tan \frac{\theta}{3}$

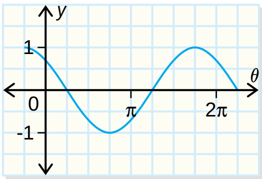
4. State the period of each function in radians.

(a) $y = \cos 4\theta$ (b) $y = \sin 6\theta$ (c) $y = \sin \frac{5}{3}\theta$ (d) $y = \cos 2\pi\theta$

(e) $y = \cos \frac{2}{5}\theta$ (f) $y = \sin \frac{2}{3}\theta$ (g) $y = \tan 3\theta$ (h) $y = \tan \frac{2\theta}{3}$

5. Match each function to its corresponding graph. Do not use technology.

(a) $y = \sin\left(\theta + \frac{\pi}{4}\right)$ (b) $y = \sin\left(\theta - \frac{\pi}{4}\right)$ (c) $y = \cos\left(\theta + \frac{\pi}{4}\right)$

i.


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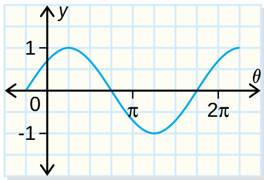

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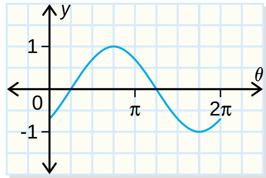
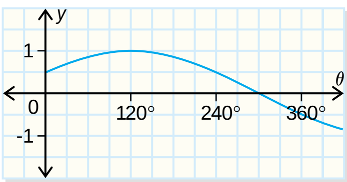


6. Match each function to its corresponding graph.

(a) $y = \sin(2\theta - 90°), 0° \leq \theta \leq 360°$

(b) $y = \sin(3\theta + 90°), 0° \leq \theta \leq 360°$

(c) $y = \sin\left(\frac{\theta}{2} + 30°\right), 0° \leq \theta \leq 360°$

i.


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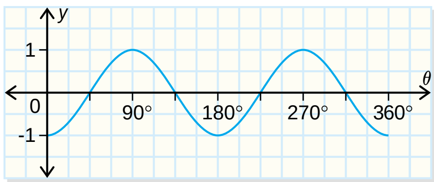

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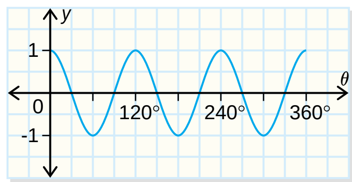


7. **Knowledge and Understanding:** State the amplitude, period, phase shift, and vertical shift for each function.

(a) $y = 3\sin(2\theta - 60°) + 1$ (b) $y = 5\cos(3\theta + 45°) - 2$

(c) $y = -2\sin\left(\frac{\theta}{3} + 15°\right) + 2$ (d) $y = \frac{1}{2}\cos\left(\frac{\theta}{2} - 7.5°\right) - 3$

(e) $y = 1 - 4\cos\left(6\theta + \frac{\pi}{3}\right)$ (f) $y = 2 + 3\sin 4\left(\theta - \frac{\pi}{2}\right)$

B

8. Sketch each graph for $0° \leq \theta \leq 360°$ by comparing it to $y = \sin\theta$. Do not use technology.

(a) $y = -\sin\theta + 2$ (b) $y = \frac{1}{2}\sin\theta - 2$ (c) $y = -2\sin\frac{\theta}{2}$

9. Sketch each graph for $0 \leq \theta \leq 2\pi$ by comparing it to $y = \cos\theta$. Do not use technology.

(a) $y = -\cos\theta + 2$ (b) $y = \frac{1}{2}\cos\theta - 2$ (c) $y = -2\cos\frac{\theta}{2}$

10. Sketch each graph for $0° \leq \theta \leq 360°$ by comparing it to $y = \tan\theta$. Do not use technology.

(a) $y = -\tan\theta$ (b) $y = \tan 2\theta$ (c) $y = \tan\frac{\theta}{2} + 1$

11. For $y = 3f(2\theta - 90°) - 1$, $f(\theta) = \sin\theta$. Evaluate y to one decimal place for each value of θ.

(a) 30° (b) 45° (c) 125° (d) −225°

12. For $y = -2f\left(3\theta + \frac{\pi}{2}\right) - 1$, $f(\theta) = \cos\theta$. Evaluate y to one decimal place for each value of θ.

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $-\frac{\pi}{6}$ (d) $-\frac{3\pi}{4}$

13. For $y = 2f(2\theta - 45°) - 1$, $f(\theta) = \tan\theta$. Evaluate y to one decimal place for each value of θ.

(a) 15° (b) 90° (c) 60° (d) −25°

14. **Communication**

(a) Explain how the graphs of $y = \sin\theta$ and $y = \cos\theta$ are alike and different?

(b) Rewrite $y = \sin\theta$ as a cosine function.

(c) Rewrite $y = \cos\theta$ as a sine function.

15. Sketch the graph of $y = -2\cos\left(\theta + \frac{\pi}{4}\right)$, $0 \leq \theta \leq 2\pi$, by comparing it to $y = \cos\theta$.

16. The graph of $y = 2\sin 3\theta$ is shifted to the right $\frac{\pi}{2}$ units and down 2 units. Write the new equation.

17. The graph of $y = -3\cos\frac{\theta}{2}$ is shifted to the left $\frac{2\pi}{3}$ units and up 1 unit. Write the new equation.

18. Sketch each graph for $-360° \leq \theta \leq 360°$. Verify the sketch using a calculator.

(a) $y = 3\sin(2\theta - 60°) + 1$ (b) $y = 5\cos(3\theta + 45°) - 2$

(c) $y = -2\sin\left(\frac{\theta}{3} + 15°\right) + 2$ (d) $y = \frac{1}{2}\cos\left(\frac{\theta}{2} - 7.5°\right) - 3$

19. Sketch each graph for $0 \leq \theta \leq 2\pi$. Verify the sketch using a graphing calculator.

(a) $y = 1 - 4\cos\left(6\theta + \frac{\pi}{3}\right)$ (b) $y = 2 + 3\sin 4\left(\theta - \frac{\pi}{2}\right)$

20. How is the graph of $y = -3\cos\left(2\theta - \frac{\pi}{4}\right) + 1$ different from the graph of $y = \cos\theta$? How is the graph of $y = -3\cos\left(2\theta - \frac{\pi}{4}\right) + 1$ different from the graph of $y = \sin\theta$?

21. **Application:** Each person's blood pressure is different. But there is a range of blood pressure values that is considered healthy. The function $P(t) = -20 \cos \frac{5\pi}{3} t + 100$ models the blood pressure, P, in millimetres of mercury, at time, t, in seconds of a person at rest.

(a) What is the period of the function? What does the period represent for an individual?

(b) How many times does this person's heart beat each minute?

(c) Sketch the graph of $y = P(t)$ for $0 \le t \le 6$.

(d) What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

22. The average monthly temperature in a region of Australia is modelled by the function $T(t) = 9 + 23 \cos \frac{\pi}{6}t$, where T is the temperature in degrees Celsius and t is the month of the year. For $t = 0$, the month is January.

(a) Prepare a table for $0 \le t \le 13$.

(b) Graph the data.

(c) Explain how to use the axis of the curve and the amplitude to determine the maximum and minimum values of the function.

(d) Determine the period of the function from the graph. Verify your answer algebraically.

(e) Verify the graph in (b) by using a graphing calculator.

(f) Explain how to sketch a similar graph using transformations of $y = \cos \theta$.

23. The function $D(t) = 4 \sin \left[\frac{360}{365}(t - 80)\right]^\circ + 12$ is a model of the number of hours of daylight, D, on a specific day, t, on the 50° of north latitude.

(a) Explain why a trigonometric function is a reasonable model for predicting the number of hours of daylight.

(b) How many hours of daylight do March 21 and September 21 have? What is the significance of each of these days?

(c) What is the significance of the number 80 in the model?

(d) How many hours of daylight do June 21 and December 21 have? What is the significance of each of these days?

(e) Explain what the number 12 represents in the model.

(f) Graph the model.

(g) What are the maximum hours of daylight? the minimum hours of daylight? On what days do these values occur?

(h) Use the graph to determine t when $D(t) = 12$. What dates correspond to t?

24. The position of the sun at sunset, north or south of due west, depends on the latitude and the day of the year. The number of degrees, P, north or south of due west for a specific latitude on a specific day, t, is modelled by the function $P(t) = 28 \sin \left(\frac{2\pi}{365}t - 1.4\right)$.

(a) What is the angle at sunset on February 28, to one decimal place? What is the angle on May 15?

(b) Graph the function.

(c) What is the maximum angle north of due west? What is the minimum angle south of due west? How could you find these angles without drawing the graph?

(d) What is the period of the function and how does it relate to the problem?

(e) What is the significance of the number 1.4 in terms of the day of the year?

25. **Check Your Understanding:** Recall $y = \sin \theta$ and $y = a \sin k(\theta + b) + d$.

(a) Explain the meaning of a, k, b, and d.

(b) Explain how to use the graph of $y = \sin \theta$ to sketch the graph of $y = 3 \sin (2\theta + 90)° + 1$.

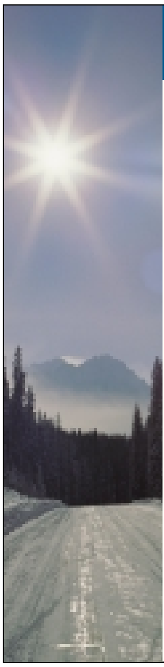
(c) Sketch the graph of $y = 3 \sin (2\theta + 90)° + 1$ following your explanation from (b). Verify your sketches using a graphing calculator.

C

26. **Thinking, Inquiry, Problem Solving:** The population, R, of rabbits and the population, F, of foxes in a given region are modelled by the functions

$R(t) = 10\,000 + 5000 \cos \frac{2\pi}{24}t$ and $F(t) = 1000 + 500 \sin \frac{2\pi}{24}t$,

where t is the time in months. Explain, referring to each graph, how the number of rabbits and the number of foxes are related.



The Chapter Problem—How Much Daylight?

In this section, you studied transformations of trigonometric functions. Apply what you learned to answer these questions about the Chapter Problem on page 404.

CP10. Refer to the graph of the original data you made earlier.

(a) Explain how the axis of the graph and the concept of vertical shift are related. What does vertical shift mean in this problem?

(b) What choices of starting position are possible when highlighting a cycle? What is the phase shift if a typical sine function were highlighted? if a typical cosine function were highlighted? What does phase shift mean in this problem?

(c) Compare the period of this data to the period of $y = \sin \theta$ or $y = \cos \theta$. How can you use this information to find the equation representing the data?