

5.3 Trigonometric Functions

The London Eye is a Ferris wheel on the south bank of the Thames River in London, England. It is 135 m high, as high as a 40-storey building, and can carry a thousand riders at once.

Sarah and her younger brother Billy visited England and rode the London Eye. During the ride, Billy kept asking, “How high are we now?” Trigonometry can be used to model the height of Sarah and Billy at different times.



Part 1: Collecting Data for the Model

Follow these steps to gather data about Sarah and Billy’s height as their capsule rotates.

Think, Do, Discuss

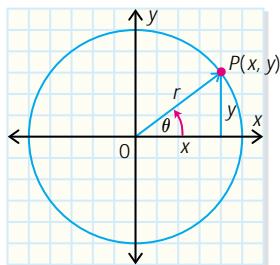
1. Draw a circle to model the London Eye. Centre the circle at the origin of the x - and y -axes. At the bottom of the circle, draw a horizontal line to represent the ground.
2. Draw points on the circle to indicate different positions on the ride. Choose points that are easy to estimate, such as points on the x -axis, the y -axis, and on the rotation angle halfway between the x - and y -axes in each quadrant.
 - (a) How high above the ground will Sarah and Billy be when they are at the top of the ride? halfway to the top? Show these points on the diagram. What are the coordinates of each point? How can you use each coordinate to determine the height above the ground?
 - (b) What acute angle corresponds to halfway between each quadrant? How many points on the ride will this angle help determine?
 - (c) Draw right triangles to the circle that include each related acute angle.
 - (d) How long is the hypotenuse of each right triangle? Explain your answer.
 - (e) What trigonometric ratio combines the acute angle and the hypotenuse to give a second side of the triangle that represents vertical distance?
 - (f) How could you use this second side length to determine Sarah and Billy’s location on the London Eye with respect to the x -axis? How could you use this second side length to find their height above the ground?

- (g) How many different positions on the Ferris wheel have you represented on your diagram? What principal angles in standard position do these positions represent? By using the angles in standard position, what assumption have you made about where Billy and Sarah get on the ride?
- (h) Determine the height above the ground, to the nearest decimal, for each angle in standard position. Create a table in your notebook to record the data.

Angle of Rotation (degrees)									
Height Above Ground (m)									

Part 2: Extending the Primary Trigonometric Ratios Beyond Triangles

Suppose Sarah and Billy were on any Ferris wheel of radius r and their location on the wheel is $P(x, y)$, as shown. θ represents the angle of rotation.



In quadrant I, the related acute angle and the angle in standard position are the same. In the right triangle for angle θ , the opposite side has length y and the adjacent side has length x . The hypotenuse of the right triangle is r , where $r = \sqrt{x^2 + y^2}$. The length r is always positive, and in quadrant I x and y are also positive.

Then,

$$\begin{aligned} \sin \theta &= \frac{\text{OPP}}{\text{HYP}} & \cos \theta &= \frac{\text{ADJ}}{\text{HYP}} & \tan \theta &= \frac{\text{OPP}}{\text{ADJ}} \\ &= \frac{y}{r} & &= \frac{x}{r} & &= \frac{y}{x} \end{aligned}$$

In general, any point, $P(x, y)$, on a circle of radius r can be expressed as a function of θ . In other words, as θ changes, corresponding changes occur in x and y , both in magnitude and sign.

Graphing the Sine and Cosine Functions

For any point $P(x, y)$ on the terminal arm of an angle θ in standard position, the trigonometric functions are defined as follows:

$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \text{ and } \tan \theta = \frac{y}{x}, x \neq 0, \text{ where } r = \sqrt{x^2 + y^2}$$

Help Sarah determine her position anywhere on a Ferris wheel with respect to the centre of the wheel.

Think, Do, Discuss

1. (a) Which of the primary trigonometric functions best links the distance of a rider on the Ferris wheel from the horizontal diameter of the wheel, the size of the Ferris wheel, and the angle of rotation? Solve the equation for this distance in terms of the other two variables.
- (b) Complete the table to show this distance for the London Eye as a function of angular rotation. Use increments of 30° for one complete revolution.

Angle of Rotation	0°	30°	60°	90°	120°
Distance on Ferris Wheel from the Horizontal Diameter of the Wheel							

- (c) Graph the distance from the horizontal diameter versus the angle of rotation.
 - (d) What is the greatest distance above the horizontal diameter? below the horizontal diameter? How can these values be used to determine the amplitude of the curve?
 - (e) What is the period of the curve?
 - (f) How would the graph look for two revolutions of the Ferris wheel? What type of model would describe the data? What equation models the distance from the horizontal diameter for any angle of rotation for the Ferris wheel?
 - (g) What is Sarah and Billy's distance from the horizontal diameter when the ride has rotated 130° ? 280° ? 420° ? 550° ? What is their height above the ground for each of these positions of the Ferris wheel if it almost touches the ground?
 - (h) Which coordinate is used to represent distance from the horizontal diameter?
 - (i) Write an equation to model their height above the ground for any angle of rotation of the Ferris wheel.
2. (a) Which primary trigonometric function best links the distance of a rider from the vertical diameter of the Ferris wheel, the size of the wheel, and the angle of rotation? Solve the equation for this distance in terms of the other two variables.
 - (b) Complete a table as in step 1(b) to show distance on the London Eye from the vertical diameter of the wheel for the given angles of rotation of the ride.
 - (c) Graph the distance from the vertical diameter versus the angle of rotation.
 - (d) What is the greatest horizontal distance to the right of the vertical diameter? to the left of the vertical diameter? How do these distances compare to those in step 1(d)? Explain. What is the amplitude of the curve?
 - (e) What is the period of the curve?
 - (f) Extend the graph to represent two revolutions of the Ferris wheel.
 - (g) What is Sarah and Billy's distance from the vertical diameter when the ride has rotated 130° ? 280° ? 420° ? 550° ?

- (h) Which coordinate is used to represent distance from the vertical diameter?
 - (i) Write an equation to model their distance from the vertical diameter at any angle of rotation.
3. (a) What x - and y -coordinates would represent Sarah and Billy's position on the London Eye at 130° ? 280° ? 420° ? 550° ? Show the coordinates on a diagram that models the Ferris wheel.
- (b) What general coordinates would represent their position on this Ferris wheel for any angle of rotation θ ? What coordinates would represent their position on any Ferris wheel or circular path of radius r ?
- (c) Draw a circle with $r = 1$ as a model of the Ferris wheel. Mark the exact coordinates of the eight positions you marked in Part 1, step 2(g) if the Ferris wheel had a radius of 1.

Graphing the Tangent Function

Sarah used the sine and cosine functions to locate her position anywhere on a circular ride. What does the graph of the tangent function look like for the Ferris wheel?

Think, Do, Discuss

1. (a) Use the data from step 1(b) and step 2(b) above to make a table to represent the tangent function. How does this data apply to the definition of the tangent function? At what points on the Ferris wheel will the tangent function not be defined? What will happen to the graph at these points?
- (b) Graph the tangent function for one revolution of the Ferris wheel.
- (c) What happens to the graph as the Ferris wheel starts its second revolution?

Part 3: Examining a Trigonometric Function in Terms of Time

Recall that Billy and Sarah's height on the Ferris wheel is a function of the angle of rotation of the wheel. How could you determine their position as a function of how long they are on the ride if the London Eye completes one revolution every 30 min?

Think, Do, Discuss

1. How many degrees does the Ferris wheel move every minute?
2. Assume Billy and Sarah start the ride halfway to its maximum height. Make a table that shows their height every 2 min for one revolution of the ride.
3. Graph height versus time.
4. Write an equation to model their height at any time, t .
5. What is their height at 17 min?

Part 4: Developing the CAST Rule

The trigonometric functions take on positive or negative signs, depending on in which quadrant the angle θ terminates. Using a circle and various points on the circumference of the circle, you can develop a rule for determining in which quadrants the primary trigonometric ratios are positive.

Think, Do, Discuss

1. Draw a circle and mark one point in each quadrant that is on the circumference.
2. What is the sign of the x -coordinate? the y -coordinate?
3. What is the sign of the radius?
4. Determine the sign of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for each point.
5. In which quadrants is
 - (a) the sine function positive?
 - (b) the cosine function positive?
 - (c) the tangent function positive?

Focus 5.3

Key Ideas

- The primary trigonometric functions for a point $P(x, y)$ on the terminal arm of angle θ in standard position are $\sin \theta = \frac{y}{r}$, $\cos \theta = \frac{x}{r}$, and $\tan \theta = \frac{y}{x}$, $x \neq 0$, where $r = \sqrt{x^2 + y^2}$.
- Any point $P(x, y)$ on a circle of radius r and rotated through angle θ can be expressed as the ordered pair $(r \cos \theta, r \sin \theta)$.
- For the unit circle, where $r = 1$, the rotation of point $P(x, y)$ through angle θ generates these graphs:

The graph of $f(\theta) = \sin \theta$ is a function.

The curve is sinusoidal.

The graph is periodic.

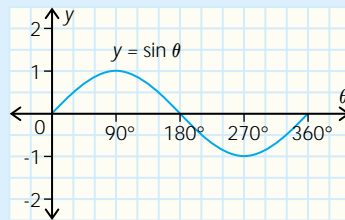
The period is 360° .

The amplitude is 1.

The maximum value of the curve is 1.

The minimum value is -1 .

The zeros of the graph for this interval are 0° , 180° , and 360° .



The graph of $f(\theta) = \cos \theta$ is a function.

The curve is sinusoidal.

The graph is periodic.

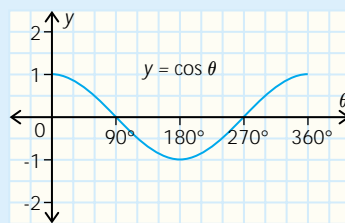
The period is 360° .

The amplitude is 1.

The maximum value of the curve is 1.

The minimum value is -1 .

The zeros of the graph for this interval are 90° and 270° .



The graph of $f(\theta) = \tan \theta$ is a function.

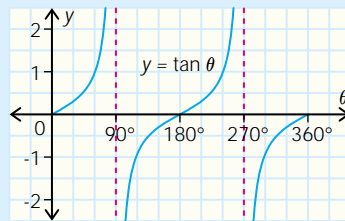
The graph is periodic.

The period is 180° .

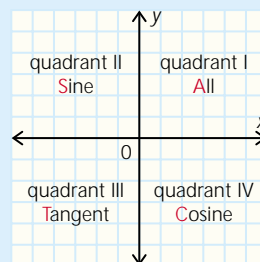
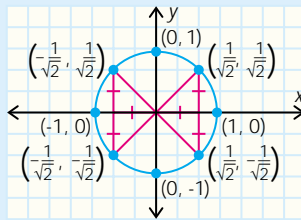
There is no maximum or minimum value.

The graph has asymptotes for this interval at $\theta = 90^\circ$ and $\theta = 270^\circ$.

The zeros of the graph for this interval are 0° , 180° , and 360° .



- A positive sign for trigonometric functions is summarized as the CAST rule: **C** indicates $\cos \theta$ is positive. **A** indicates all are positive. **S** indicates $\sin \theta$ is positive. **T** indicates $\tan \theta$ is positive.
- The rotation of a point, in 45° increments, about the unit circle generates useful coordinates for graphing trigonometric functions.



Example 1

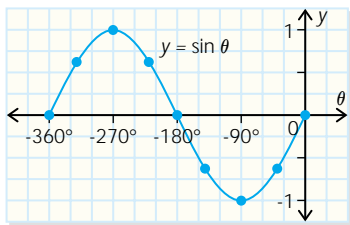
- Construct a table for $y = \sin \theta$, where $-360^\circ \leq \theta \leq 0^\circ$. Use intervals of 45° and the unit circle to draw the graph by hand. Round to one decimal place.
- Extend the graph to include $0^\circ \leq \theta \leq 720^\circ$.
- State the location of all maximum and minimum values for $-360^\circ \leq \theta \leq 720^\circ$.

Solution

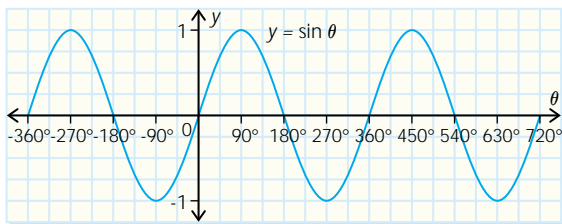
- Make a table in 45° increments using the unit circle.

θ	-360°	-315°	-270°	-225°	-180°	-135°	-90°	-45°	0°
$\sin \theta = \frac{y}{r}$	$\frac{0}{1} = 0$	$\frac{(\frac{1}{\sqrt{2}})}{1} \doteq 0.7$	$\frac{1}{1} = 1$	$\frac{(\frac{1}{\sqrt{2}})}{1} \doteq 0.7$	$\frac{0}{1} = 0$	$\frac{-\frac{1}{\sqrt{2}}}{1} \doteq -0.7$	$-\frac{1}{1} = -1$	$\frac{-\frac{1}{\sqrt{2}}}{1} \doteq -0.7$	$\frac{0}{1} = 0$

Draw the graph.



- (b) The graph is sinusoidal and has a period of 360° . Extend the pattern to 720° .



- (c) The maximum values are at -270° , 90° , and 450° . The minimum values are at -90° , 270° , and 630° . These can be read directly from the graph.

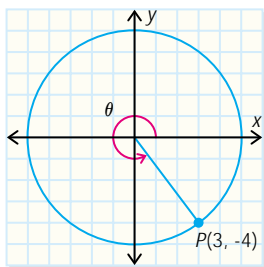
Example 2

Point $P(3, -4)$ is on the terminal arm of an angle in standard position.

- (a) What are the values of the primary trigonometric functions?
 (b) What is the measure of the principal angle θ to the nearest degree?

Solution

- (a) Draw a sketch.



Determine the values of x , y , and r .

$$x = 3, y = -4, \text{ and } r = \sqrt{(3)^2 + (-4)^2} = 5$$

Using these values, then

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{y}{x} \\ &= -\frac{4}{5} & &= \frac{3}{5} & &\doteq -\frac{4}{3} \end{aligned}$$

- (b) To evaluate θ , select any one of the primary trigonometric functions and solve for θ . Using $\sin \theta = -\frac{4}{5}$ gives,

$$\theta = \sin^{-1}\left(-\frac{4}{5}\right)$$

$$\theta \doteq -53^\circ$$

From the sketch, θ is clearly not 53° . This angle is the related acute angle. In this case, $\theta = 360^\circ - 53^\circ$ or 307° .

Example 3

For $\tan \theta = -\frac{7}{24}$, where $0^\circ \leq \theta \leq 360^\circ$,

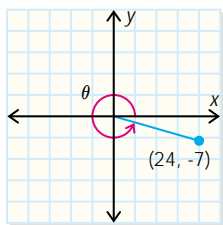
- in which quadrant is it possible for θ to terminate?
- determine the other primary trigonometric functions for θ
- evaluate θ to the nearest degree

Solution

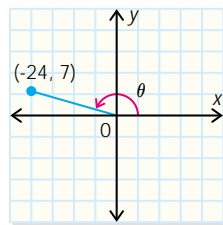
- (a) Since $\tan \theta = \frac{y}{x}$ and $\tan \theta = -\frac{7}{24}$, then $\frac{y}{x} = -\frac{7}{24}$.

There are two possibilities: $x = 24$ and $y = -7$, or $x = -24$ and $y = 7$.

For the ordered pair $(24, -7)$,
the angle terminates in quadrant IV.



For the ordered pair $(-24, 7)$,
the angle terminates in quadrant II.



- (b) For $(x, y) = (24, -7)$,

$$\begin{aligned} r &= \sqrt{(24)^2 + (-7)^2} \\ &= \sqrt{576 + 49} \\ &= 25 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \text{and} & & \cos \theta &= \frac{x}{r} \\ &= -\frac{7}{25} & & & &= \frac{24}{25} \end{aligned}$$

- For $(x, y) = (-24, 7)$,

$$\begin{aligned} r &= \sqrt{(-24)^2 + (7)^2} \\ &= \sqrt{576 + 49} \\ &= 25 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \text{and} & & \cos \theta &= \frac{x}{r} \\ &= \frac{7}{25} & & & &= -\frac{24}{25} \end{aligned}$$

The solutions for both points can be combined and written in the simplified form

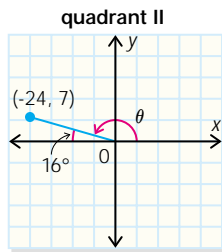
$$\sin \theta = \pm \frac{7}{25} \quad \text{and} \quad \cos \theta = \pm \frac{24}{25}.$$

- (c) Select any one of the trigonometric functions to determine the measure of θ .

Using $\sin \theta = \frac{7}{25}$, then $\theta = \sin^{-1}\left(\frac{7}{25}\right)$ or about 16° .

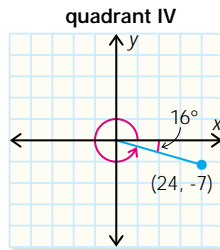
Clearly, 16° is not in quadrant II or in quadrant IV.

16° is the related acute angle.



Then, for $0^\circ \leq \theta \leq 360^\circ$,

$$\begin{aligned}\theta &= 180^\circ - 16^\circ \\ &= 164^\circ\end{aligned}$$



Then, for $0^\circ \leq \theta \leq 360^\circ$,

$$\begin{aligned}\theta &= 360^\circ - 16^\circ \\ &= 344^\circ\end{aligned}$$

Example 4

The height, h , of a basket on a water wheel at time t is given by $h(t) = \sin(6t)^\circ$, where t is in seconds and h is in metres.

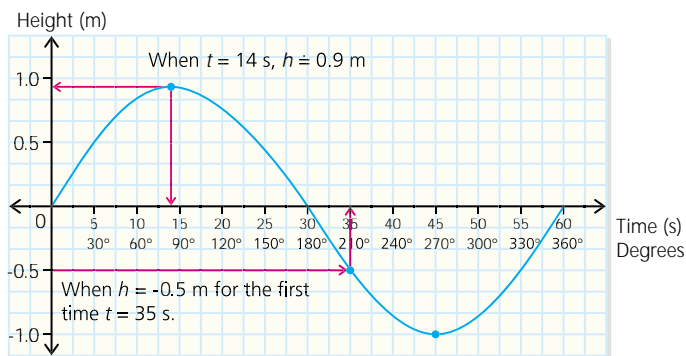
- How high is the basket at 14 s?
- When will the basket first be 0.5 m under water?

Solution

- The values of h and t can be interpolated from a graph. Prepare a table. In this case, 5-s intervals were used, although the interval size could be different.

t (seconds)	0	5	10	15	20	25	30	35	40	45	50	55	60
$(6t)^\circ$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$h(t) = \sin(6t)^\circ$ (metres)	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Interpolating from the graph gives a value of about 0.9 m, as shown.



The question could also be answered by substituting into the equation.

Then,

$$\begin{aligned} h(14) &= \sin(6 \times 14)^\circ \\ &= \sin 84^\circ \\ &\approx 0.995 \end{aligned}$$

Substitution gives a more accurate answer in this case. At 14 s, the height is almost 1 m.

- (b) A height of 0.5 m under water corresponds to a height of -0.5 m in the model. Therefore,

$$\begin{aligned} h(t) &= \sin(6t)^\circ \\ -0.5 &= \sin(6t)^\circ \end{aligned} \quad \text{Interpolation shows that } \sin 210^\circ = -0.5.$$

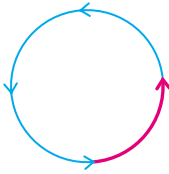
The basket will be 0.5 m under water for the first time at 35 s.

Practise, Apply, Solve 5.3

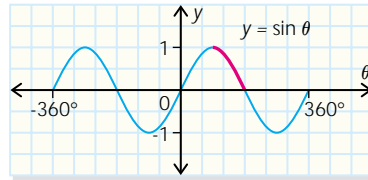
A

1. Point $P(x, y)$ is on the terminal arm of an angle θ in standard position. It could be anywhere on the highlighted part of the unit circle as shown. Match each unit circle to its graph of $y = \sin \theta$.

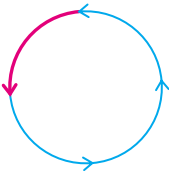
(a)



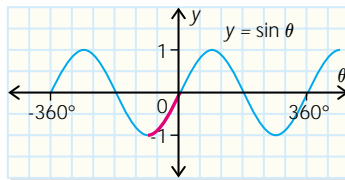
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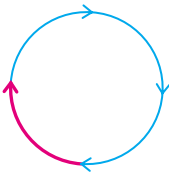
(b)



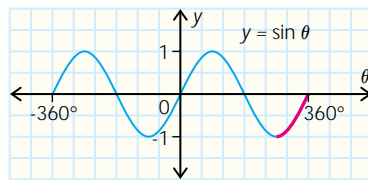
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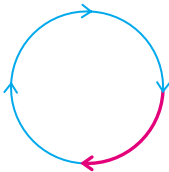
(c)



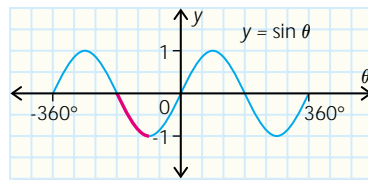
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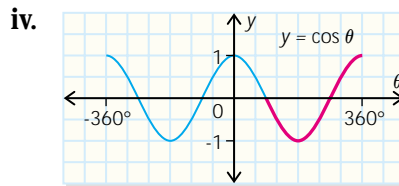
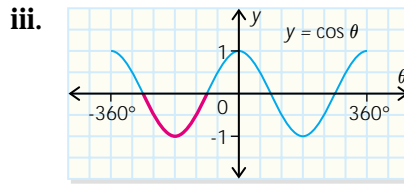
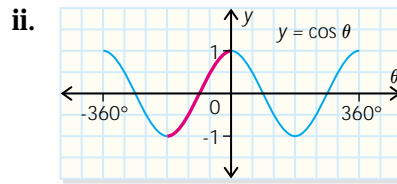
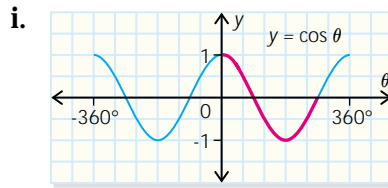
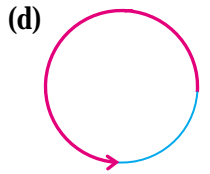
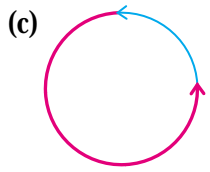
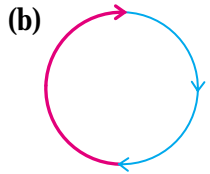
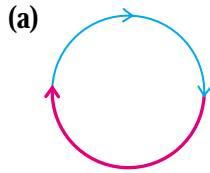
(d)



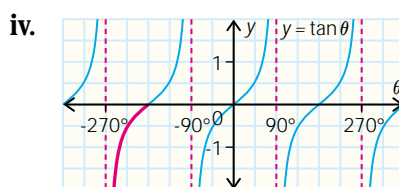
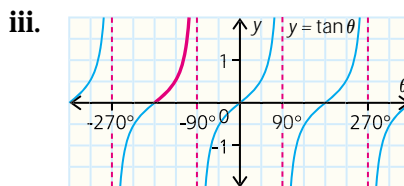
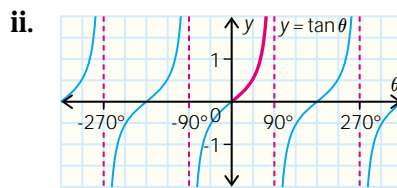
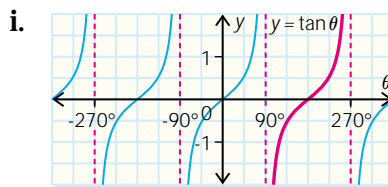
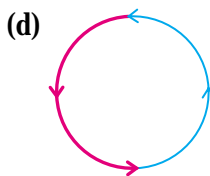
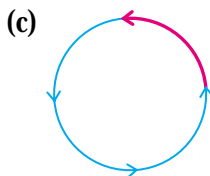
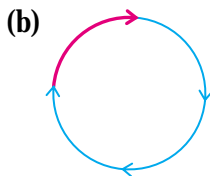
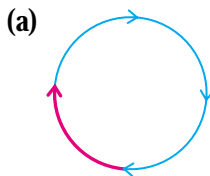
iv.



2. Repeat question 1 for the graph of $y = \cos \theta$.



3. Repeat question 1 for the graph of $y = \tan \theta$.



4. Use the CAST rule to state the sign of each value. Check using a calculator.
- (a) $\tan 15^\circ$ (b) $\sin 120^\circ$ (c) $\cos 135^\circ$ (d) $\tan 110^\circ$
 (e) $\cos 205^\circ$ (f) $\tan (-15^\circ)$ (g) $\cos 120^\circ$ (h) $\sin (-45^\circ)$
5. Determine the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ at each point $P(x, y)$ on the terminal arm of an angle θ in standard position.
- (a) $(-3, -4)$ (b) $(3, 4)$ (c) $(5, 12)$ (d) $(-12, 5)$
 (e) $(7, -24)$ (f) $(-7, 24)$ (g) $(0, -1)$ (h) $(-1, 0)$
6. Each point is on the terminal arm of an angle θ in standard position. Find the principal value of θ to the nearest degree.
- (a) $(5, 11)$ (b) $(9, -2)$ (c) $(-4, -7)$ (d) $(-5, 9)$

B

7. Convert each point, $P(x, y)$, to the ordered pair $P(r \cos \theta, r \sin \theta)$. Round all values of r to one decimal place and all values of θ to the nearest degree.
- (a) $(4, 6)$ (b) $(-3, 7)$ (c) $(10, -23)$
8. **Knowledge and Understanding:** Point $(-3, -4)$ is on the terminal arm of an angle α (alpha) in standard position. Verify that $(\sin \alpha)^2 + (\cos \alpha)^2 = 1$.
9. Determine $\sin \theta$ to three decimal places when $\tan \theta = \frac{1}{2}$ and θ is an angle in the first quadrant.
10. Determine $\cos \alpha$ to three decimal places when $\sin \alpha = \frac{6}{\sqrt{61}}$ and α is an angle in the second quadrant.
11. Given that $\cos \alpha = -\frac{8}{17}$ and that $0^\circ \leq \alpha \leq 360^\circ$, find two values of α , to two decimal places.
12. **Application:** Given that $\sin \alpha = \frac{15}{17}$ and $-360^\circ \leq \alpha \leq 360^\circ$, find all values of α to two decimal places.
13. Consider the function $f(\theta) = \cos \theta$.
- (a) Complete the table using the unit circle and sketch the graph.

θ	-360°	-270°	-180°	-90°	0°	90°	180°	270°	360°
$f(\theta)$									

- (b) State the coordinates of the maximum and minimum values of $f(\theta) = \cos \theta$ within the domain of the table.
- (c) What are the coordinates of the zeros of the function within this domain?
- (d) Show that $f(\theta) = f(-\theta)$ for all values of θ in the table.
- (e) Explain why $f(\theta) = f(-\theta)$ for all θ .

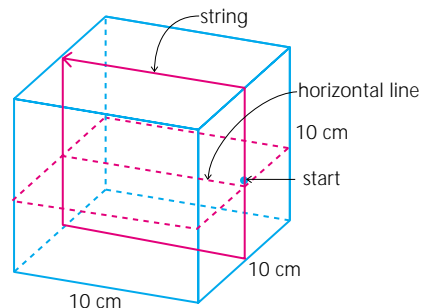
14. Consider the function $f(\theta) = \tan \theta$.
- (a) Complete the table and sketch the graph.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$f(\theta)$													

- (b) What are the coordinates of the zeros of the function within the domain of the table?
- (c) Explain why the domain $\theta = n^\circ$, $n \in \mathbf{R}$, is not sufficient for $f(\theta) = \tan \theta$. What restrictions must be added?
- (d) State all zeros of the function.
- (e) Extend the table to $\theta = -360^\circ$ using the same intervals. Show that $f(\theta) = -f(-\theta)$ for all values of θ in the table.
- (f) Explain why $f(\theta) = -f(-\theta)$ for all θ .
15. Evaluate $y = \cos \theta$ for $0^\circ \leq \theta \leq 540^\circ$ when $y = -0.7$. Answer to the nearest degree.
16. Evaluate $y = \sin \theta$ for $-90^\circ \leq \theta \leq 540^\circ$ when $y = -0.3$. Answer to the nearest degree.
17. (a) Evaluate $h(t) = \cos (20t)^\circ$ for $t = 3$.
 (b) What is the value of t when $h(t) = 0.3$ for $0 \leq t \leq 18$?
18. (a) Evaluate $h(t) = 4 \sin (30t)^\circ$ for $t = 10$.
 (b) What is the value of t when $h(t) = 3.2$ for $0 \leq t \leq 12$?
19. The vertical distance in metres of a rider with respect to the horizontal diameter of a Ferris wheel is modelled by $h(t) = 5 \cos (18t)^\circ$, where t is the number of seconds.
- (a) To one decimal place, what is the rider's vertical distance with respect to the horizontal diameter of the wheel when $t = 8$ s? 16 s? 30 s?
- (b) When is the rider first at 4.5 m? -3.2 m?
- (c) When is the third time the rider is at -2.5 m?
20. **Communication:** Draw a tangent graph that completes a full cycle from -90° to 90° . Extend the domain to 630° . Show all asymptotes and label the points where the asymptotes cross the horizontal axis. Write these points as a sequence.
21. (a) Determine the sign of each primary trigonometric function for θ terminating in quadrant I. Repeat for quadrants II, III, and IV.
 (b) Which trigonometric functions are positive in each quadrant?
 (c) Graph each trigonometric function for $0^\circ \leq \theta \leq 360^\circ$. Shade the positive part of the graph and link the graph to your answer for (b).
22. **Check Your Understanding:** Explain why $y = \sin \theta$ and $y = \cos \theta$ have maximum and minimum values, but $y = \tan \theta$ does not.

C

23. **Thinking, Inquiry, Problem Solving:** A string is wrapped once in a counterclockwise direction around a cube with 10 cm sides as shown.

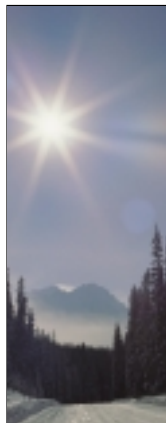


- Graph the location of the string around the cube with respect to its distance above or below the horizontal line indicated.
- How much string is used to go once around the cube? What is the period of the graph? How are the perimeter of the cube and the period length of the graph related?
- Extend the graph to show the string wrapped around the cube two times. How much string is used?
- What is the minimum value of the graph? What is the least amount of string needed for this minimum value?

24. Determine all values of θ for which $\sin \theta = \cos \theta$, for $-360^\circ \leq \theta \leq 360^\circ$.

25. The secondary trigonometric functions are the reciprocals of $\sin \theta$, $\cos \theta$, and $\tan \theta$. The reciprocal of $\sin \theta$ is the **cosecant** and is written **csc θ** . The reciprocal of **cos θ** is **secant θ** and is written **sec θ** . The reciprocal of **tan θ** is the cotangent θ and is written **cot θ** .

- Write each secondary trigonometric function in terms of its primary trigonometric function.
- Write each secondary trigonometric function in terms of a point $P(x, y)$ on the terminal arm of an angle θ in standard position. State all restrictions.
- Make a table for each secondary function and then sketch its graph for $0^\circ \leq \theta \leq 360^\circ$.
- Show how the restrictions on the domain of each function appear on the graph.
- State the domain and range of each secondary trigonometric function.



The Chapter Problem — How Much Daylight?

In this section, you studied trigonometric functions. Apply what you learned to answer these questions about the Chapter Problem on page 404.

- Sketch the graph for all 25 months in the table.
- Highlight one complete cycle for the graph that resembles a typical sine function. Highlight a cycle that resembles a typical cosine function.
- What type of periodic function could be used to model the relationship between time in months and average monthly hours of daylight?