Chapter 4 Review

Quadratic Functions and Rational Expressions

Check Your Understanding

1. Describe three different forms for the equation of a quadratic function. Explain the importance and uses of each form. Describe the algebraic methods for moving from one form to another.
2. What shape is the graph of a quadratic function? What information do the coordinates of the vertex of the graph provide? How can you determine the vertex coordinates when the equation is in standard form? in factored form?
3. How are the zeros of a function related to the graph of the function? How many zeros does a quadratic function have? Describe how you would determine the number of zeros from the equation of a quadratic function in (a) factored form, (b) vertex form, and (c) standard form.
4. Describe the sets of numbers that make up the set of real numbers. What are complex numbers and how are they related to real numbers? What are complex conjugates and how are they connected to quadratic equations?
5. How do you add or subtract complex numbers? How do you multiply complex numbers? What happens when you add complex conjugates? What happens when you multiply complex conjugates? How do you divide complex numbers? Give an example for each.
6. What is inverse variation? Give an example of two quantities that vary inversely and write an equation that describes the relationship between them. State the domain and the range and sketch the graph of this relation.
7. How can you use the graph of a function to obtain the graph of its reciprocal function? Explain the steps using an example.
8. What is a rational function? How are the graphs of reciprocal and rational functions different from the graphs of the other functions you have studied? Describe their features with examples.
9. Explain how you would simplify a rational expression. Include a discussion of restrictions. Why must you state the restrictions before reducing an expression?
10. When you add or subtract rational expressions, what steps do you take to find the lowest common denominator? Explain with an example.
11. Write an example of a polynomial expression, state its degree, and explain how you determined the degree. What special names are given to polynomials with one, two, or three terms? Is your polynomial one of these special types? Explain.

Review and Practice

4.1 Skill Builder: Extending Algebra Skills: Completing the Square

1. What is the vertex form of a quadratic function? How does this form relate to completing the square? What other equivalent forms may a quadratic function have? Give an example of each form.

2. Describe the steps for changing a quadratic function in standard form to vertex form by completing the square.

3. Express each function in vertex form by completing the square.

(a) $f(x) = x^2 - 6x + 7$
(b) $f(x) = x^2 - 5x + 1$
(c) $f(x) = 2x^2 + 7x - 3$
(d) $f(x) = -4x^2 - 3x + 2$
(e) $f(x) = -1.6x^2 - 2.4x - 1$
(f) $f(x) = \frac{2}{3}x^2 - \frac{4}{9}x + 2$

4. For each quadratic function, find the coordinates of the vertex by completing the square, state the domain and the range of $f(x)$, and sketch the graph.

(a) $f(x) = x^2 + 8x + 13$
(b) $f(x) = -2x^2 + 5x$
(c) $f(x) = \frac{1}{2}x^2 - 5x + 15$

5. The height, h, in metres, of a baseball after Bill hits it with a bat is described by the function $h(t) = 0.8 + 29.4t - 4.9t^2$, where t is the time in seconds after the ball is struck. What is the maximum height of the ball? At what time does the ball reach this maximum height? How high above the ground was the ball when it was hit?

4.2 Maximum and Minimum Values of Quadratic Functions

6. What features of the graph of a quadratic function are most easily determined from the function in

(a) in standard form?
(b) in factored form?
(c) in vertex form?

Give an example for each form.

7. How are the graphs of $f(x) = ax^2 + bx$ and $g(x) = ax^2 + bx + c$ related? How does this help you find the maximum or minimum value for each function?

8. How can you tell from the equation of a quadratic function whether the function has a maximum or minimum value? For each of the following functions, and without changing the form of the equation,

i. state whether the function has a maximum or a minimum value
ii. find the value of x that produces the maximum or minimum value
iii. find the maximum or minimum value of the function

(a) $f(x) = -2(x - 1)^2 + 6$
(b) $f(x) = (x - 3)(x + 7)$
(c) $h(t) = -5t^2 + 20t$
(d) $g(x) = -0.1(3.2 - x)(2.1 + x)$
(e) $f(x) = 3.2(x - 1.1)^2 - 4.5$
(f) $C(x) = 0.04x^2 + 18x + 198$

9. The demand function for a new product is $p(x) = -5x + 22$, where x is the number of items sold in thousands and p is the price in dollars. The cost function is $C(x) = 3x + 15$.

(a) State the corresponding revenue function.
(b) Find the corresponding profit function.
(c) Complete the square to find the value of x that will maximize profits.
(d) Find the break-even quantities.
(e) Sketch the graph of the profit function.

10. It costs a bus company \$225 to run a minibus on a ski trip, plus \$30 per passenger. The bus has seating for 22 passengers, and the company charges \$60 per fare if the bus is full. For each empty seat, the company has to increase the ticket price by \$5. How many empty seats should the bus run with to maximize profit from this trip?

4.3 Zeros of Quadratic Functions

11. Sketch the graph of a quadratic function that has no zeros. Explain how you know that the function you have sketched has no zeros. Repeat for a quadratic function that has (a) one zero, and (b) two zeros.

12. What is the discriminant and how may you use it to find the number of zeros for the function $f(x) = ax^2 + bx + c$? Give an example of a function in standard form with (a) no zeros, (b) one zero, and (c) two zeros.

13. For a quadratic function in the form $y = a(x - h)^2 + k$, how may you use the values of a and k to find the number of zeros? Give an example of a function written in vertex form with (a) no zeros, (b) one zero, and (c) two zeros.

14. Without drawing the graph,

i. find the number of zeros for each function

ii. indicate whether the graph touches the x-axis at one point, intersects the x-axis at two points, or does not meet the x-axis

(a) $f(x) = 3x^2 + 4x + 1$

(b) $g(x) = -2(x + 3.6)^2 + 4.1$

(c) $h(x) = -5(x + 1.3)^2$

(d) $m(x) = 4(x + 1)^2 + 0.5$

(e) $n(x) = -x^2 - 6x - 9$

(f) $p(x) = -3.2(x + 1.1)(x - 2.4)$

15. Find the value(s) of k so that the graph of each function has

i. two x-intercepts ii. one x-intercept iii. no x-intercepts

(a) $f(x) = kx^2 - 8x + 2$

(b) $f(x) = 2x^2 + kx + 50$

(c) $f(x) = 9x^2 + kx + 4$

(d) $f(x) = x^2 + (k + 1)x + 1$

4.4 Introducing Complex Numbers

16. Draw a diagram to show how the sets of numbers used in mathematics relate to each other and are connected to the set of real numbers and to the set of complex numbers.

17. How are complex numbers different from real numbers? What form do complex numbers take and when do you use them?

18. Under what circumstances will a quadratic equation have complex roots? How are these roots connected?

19. Identify *all* sets of numbers to which each number belongs.

(a) 0.85

(b) -7

(c) $\sqrt{-3}$

(d) $\frac{5}{9}$

(e) $2 + 5i$

(f) $\sqrt{5}$

(g) $2.\overline{45}$

(h) $\frac{-3}{7}$

(i) 0

(j) $\sqrt{9}$

20. Write the complex conjugate of each complex number.

(a) $2 - i$

(b) $1 + 3i$

(c) $-4 - 5i$

(d) $-5 + 6i$

(e) $13 - 13i$

(f) $3 + \sqrt{3}i$

21. Solve each quadratic equation for x, where $x \in \mathrm{C}$. Round to two decimal places, where necessary.

(a) $x^2 + 81 = 0$

(b) $-2x^2 = 72$

(c) $x^2 - 2x + 3 = 0$

(d) $x^2 - 4x + 20 = 0$

(e) $2x^2 = 3x - 2$

(f) $3x(x + 4) = -14$

4.5U Skill Builder: Adding, Subtracting, and Multiplying Complex Numbers

22. Give an example to show how to add or subtract complex numbers. What happens when you add two complex conjugates?

23. Give another example to show how to multiply complex numbers. What happens when you multiply two complex conjugates?

24. Simplify each of the following.

(a) $(3i)^2$ (b) i^5

(c) $(2 - i)(5 + 2i)$ (d) $(-3 - 4i)(2 - 3i)$

(e) $(-1 + 7i)(-1 - 7i)$ (f) $-i(3 - 5i)$

(g) $(-1 + 4i)^2$ (h) $\frac{1}{(2 + i)(2 - i)}$

(i) $(7 - 2i)^2 - (2 - 3i)^2 + 2i$ (j) $(5 + 3i) + (8 - 2i) - (6 - 3i)$

25. Find $z + \overline{z}$, $z - \overline{z}$, and $z\overline{z}$ for each value of z.

(a) $z = 3 - i$ (b) $z = 3 + 4i$ (c) $z = -5 + 2i$

26. If $f(x) = -2x^2 - x + 2$, then find $f(3 - 2i)$.

27. In each case, one root of a quadratic equation is given. Find the other root and write a quadratic equation that has these roots. Express the answer in the form $ax^2 + bx + c = 0$.

(a) $3i$ (b) $1 - 2i$ (c) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

4.6–4.7 Reciprocal Functions

28. Explain why the function $f(x) = \frac{1}{x}$ is said to display inverse variation. What special features does the graph of this function have? What is the name for this type of graph? Describe the domain and the range for $f(x) = \frac{1}{x}$.

29. What properties of the graph of $y = f(x)$ can help you graph $y = \frac{1}{f(x)}$?

30. (a) Which of the following tables display inverse variation? Explain your answer.

i.

x	y
2	30
3	20
5	12
8	7.5

ii.

x	y
12	240
9	320
5	576
2	1440

iii.

x	y
2	7
4	31
6	71
8	127

(b) Write an equation for each relation.

31. The time a car takes to accelerate to a certain speed from rest and the acceleration vary inversely. The car takes 5 s to reach a certain speed when the acceleration is 12 m/s^2. Find the time the car takes to reach the same speed when the acceleration is 15 m/s^2. Write an equation to model the relation between time and acceleration. Sketch the graph of the relation.

32. In each case, graph $y = f(x)$ and use it to graph $y = \frac{1}{f(x)}$ on the same axes.

(a) $f(x) = x^2 - 1$ (b) $f(x) = (x - 3)^2 - 4$ (c) $f(x) = (x + 1)^2$

(d) $f(x) = x^2 + 3$ (e) $f(x) = -(x + 2)^2 + 3$ (f) $f(x) = \sqrt{2 - x} - 4$

33. For each function, draw $f(x)$, $\frac{1}{f(x)}$, and $f^{-1}(x)$ on the same axes. Also, state the domain and the range.

(a) $f(x) = 2x - 1$ (b) $f(x) = x^2 - 3$ (c) $f(x) = \sqrt{x - 2} - 3$

4.8 Simplifying Rational Expressions

34. What is a rational expression? How does factoring help when simplifying a rational expression?

35. When is a rational expression not defined? How does this condition affect simplifying rational expressions?

36. Simplify and state any restrictions.

(a) $\frac{3x + 9}{-5x - 15}$ (b) $\frac{48p^3q^2}{-18pq^3}$ (c) $\frac{16x^2 - 9}{4x^2 + x - 3}$

(d) $\frac{a^2 - 2a - 8}{a^2 - a - 12}$ (e) $\frac{10x^2 - 11x - 6}{10x^2 + 19x + 6}$ (f) $\frac{x^2 - 6x + 9}{9 - x^2}$

(g) $\frac{2x^2 - 5xy + 2y^2}{y^2 - 3xy + 2x^2}$

37. (a) The concentration of pain-killing drug in the bloodstream t hours after it is taken orally is given by the expression $\frac{5t^5 + 10t^3}{t^6 + 4t^4 + 4t^2}$. Find the concentration, to one decimal place, after $\frac{1}{2}$ h, 1 h, and 2 h.

(b) The drug may be taken again once the concentration drops below 1. How long does it take for the concentration to drop to this level?

4.9 Multiplying and Dividing Rational Expressions

38. Explain the steps for multiplying rational expressions. Give reasons for each step. What other steps are needed for dividing rational expressions?

39. Why are there extra restrictions when dividing rational expressions?

40. Simplify and state any restrictions.

(a) $\frac{6x}{8y} \times \frac{2y^2}{3x}$ (b) $\frac{10m^2}{3n} \times \frac{6mn}{20m^2}$ (c) $\frac{2ab}{5bc} \div \frac{6ac}{10b}$ (d) $\frac{5p}{8pq} \div \frac{3p}{12q}$

41. Simplify and state any restrictions.

(a) $\frac{3}{x^2 - 9} \div \frac{3x - 6}{x - 3}$

(b) $\frac{1}{10a - 2a^2} \div \frac{a}{4a - 20}$

(c) $\frac{3}{y^2 - 2y - 15} \times \frac{y^2 - 9}{6y - 18}$

(d) $\frac{x^2 + 4x - 21}{x^2 - 6x - 16} \times \frac{x^2 - 8x + 15}{x^2 + 9x + 14}$

42. If $f(x) = \frac{x^2 - 9}{3x - x^2}$ and $g(x) = \frac{x^2 - 5x + 6}{2x^2 - 4x}$, then find

(a) $f(1)$ (b) $f(-1)$ (c) $g(5)$ (d) $g(-2)$

(e) $f(3)$ (f) $f(-3)$ (g) $\frac{f(x)}{g(x)}$

For what values of x is each function not defined?

43. Simplify and state any restrictions.

(a) $\frac{x^2}{2xy} \times \frac{x}{2y^2} \div \frac{(3x)^2}{xy^2}$

(b) $\frac{x^2 - 5x + 6}{x^2 - 1} \times \frac{x^2 - 4x - 5}{x^2 - 4} \div \frac{x - 5}{x^2 + 3x + 2}$

(c) $\frac{1 - x^2}{1 + y} \times \frac{1 - y^2}{x + x^2} \div \frac{y^3 - y}{x^2}$

(d) $\frac{x^2 - y^2}{4x^2 - y^2} \times \frac{4x^2 + 8xy + 3y^2}{x + y} \div \frac{2x + 3y}{2x - y}$

(e) $\frac{a^2 + 5ab + 6b^2}{a^2 - 5ab + 6b^2} \times \frac{a^2 - 2ab - 3b^2}{a^2 + ab - 2b^2} \div \frac{a^2 + 4ab + 3b^2}{a^2 - ab - 2b^2}$

4.10U Skill Builder: Dividing Complex Numbers

44. What property of complex conjugates do you use to simplify reciprocals and quotients of complex numbers?

45. Explain how to divide complex numbers. Use an example to show the steps.

46. For each complex number,

i. write the complex conjugate

ii. write the reciprocal

iii. express each reciprocal in the form $a + bi$, where a and b are real

(a) $2 - i$ (b) $-1 + i$ (c) $\sqrt{2} - \sqrt{2}i$

(d) $-3 + \sqrt{3}i$ (e) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

47. Express in the form $a + bi$.

(a) $\frac{4}{1 - i}$ (b) $\frac{5}{2 + i}$ (c) $\frac{6i}{1 + i}$ (d) $\frac{8}{1 - 3i}$

(e) $\frac{3 - i}{2 + i}$ (f) $\frac{5i - 2}{2 + 3i}$ (g) $\frac{5 - i}{i}$ (h) $\frac{-3 + 4i}{-i}$

(i) $\frac{a + bi}{a - bi}$ (j) $\frac{(1 - i)(3 + 2i)}{1 + i}$ (k) $\frac{(4 - i)(2 + 3i)}{(2 - i)^2}$

48. (a) If $z = \sqrt{3} + i$, then express $z + \frac{9}{z}$ in its simplest form.

(b) If $z = x + yi$, then find the real and imaginary parts of $z + \frac{1}{z}$.

49. Solve for z and express in the form $a + bi$.

(a) $(4 - 3i) = z(1 + i)$ (b) $6z - z(4 - i) = -2 + 6i$

4.11 Adding and Subtracting Rational Expressions

50. When two rational expressions are added or subtracted, what must be true about the two denominators if the LCD is not the product of them? Give an example. Then find the sum or difference, showing the steps for finding the LCD.

51. When you are finding a sum or difference of rational expressions, why is it important to note restrictions on the variables *before* you reduce the resulting expression?

52. Simplify. Remember to state any restrictions, and reduce the answer to lowest terms.

(a) $\frac{4}{5x} - \frac{2}{3x}$

(b) $\frac{3}{ab^2} + \frac{5}{a^2b}$

(c) $\frac{5}{x+1} - \frac{2}{x-1}$

(d) $\frac{4x}{(x+3)^2} - \frac{3}{(x+3)}$

(e) $\frac{1}{x^2+3x-4} + \frac{1}{x^2+x-12}$

(f) $\frac{1}{x^2-5x+6} - \frac{1}{x^2-9}$

(g) $\frac{x^2-4y^2}{x^2+2xy} + \frac{x^2-2xy+4y^2}{2xy}$

(h) $\frac{1}{x^2+2x-3} + \frac{1}{x^2-1} - \frac{2}{x^2+4x+3}$

(i) $\frac{x^2-1}{(x+1)^2} - \frac{x^2-5x+6}{x^2-4x+4} + \frac{x+3}{x^2+4x+3}$

(j) $\frac{a-b}{ab} - \frac{a-2b}{ab+b^2} - \frac{2a+b}{a^2-ab}$

53. Find the zeros of each function. First rewrite the equation in a different form.

(a) $f(x) = 4x - 12 + \frac{9}{x+2}$

(b) $f(x) = 9x - 15 + \frac{40}{x+3}$

54. Solve for x.

(a) $\frac{3}{x-5} + \frac{2x}{x-3} = 5$

(b) $\frac{3x}{x-1} + 4 = \frac{x}{x+1}$

(c) $\frac{1}{1-x} - \frac{1}{1+x} = \frac{4x}{1+x^2}$

(d) $\frac{5}{x+2} + \frac{x(x+3)}{x^2-4} = \frac{x}{x-2}$

55. In a motorcycle race, one lap of the course is 650 m. At the start of the race, Genna sets off 4 s after Tom does, but she drives her motorcycle 5 m/s faster and finishes the lap 2.5 s sooner than he does.

(a) Find the speed at which each of them is driving.

(b) Find the time taken by each of them to cover the distance.

4.12 Skill Builder: Extending Algebra Skills: Working with Polynomials

56. Write three examples of polynomials and state the degree of each one. What is special about the exponents in a polynomial? How do you find the degree of a polynomial?

57. Use your polynomials from question 56 to show how polynomials are added, subtracted, and multiplied.

58. Create an expression that contains a sum and a difference and involves all of the following: (a) a product of a binomial and a trinomial, (b) a product of three binomials, and (c) the square of a trinomial.

Simplify your expression. Then state the degree of the resulting polynomial.

59. Simplify.

(a) $(5x^4 - x^2 - 2) - (x^4 - 2x^3 + 3x^2 - 5)$

(b) $(-x^2 + x - 6) + (x^2 - 4x + 1) - (2x^2 - x - 5)$

(c) $-3x(x^2 - 2x - 4) - x(x^2 - 3x - 2)$

(d) $4x^2(2x^3 - x + 3) - x^3(3x^3 + 5x - 1)$

(e) $x[2x - (3x - 4)] + 4x[2x - (3 - x)]$

(f) $x[2x - (5x - 4)] + 4x[x - (6 - x)]$

60. Simplify.

(a) $(x^2 + 4)(x^2 - 2x + 3)$

(b) $3x(3x^2 - 1)^2 - 2x(1 - x^2)^2$

(c) $(t - 2)(3t - 1)(4t + 3) - (2 - t)^2(t + 5)$

(d) $(x - 3)^2(x + 1)^2 - (x + 2)^2(x - 4)^2$

(e) $(3x - 2)(x^3 + 4x^2 - 2x - 1)$

(f) $(a^2 + 4a - 3)(3a^2 - 2a + 4)$

(g) $(x^2 - 3x + 2)^2$

(h) $(x - 4)^2(x + 1)^2$

61. Prove that the triangle with sides $\frac{1}{2}(n^2 + 1)$, $\frac{1}{2}(n^2 - 1)$, and n contains a right angle.

Chapter 4 Summary

In this chapter, you reviewed and extended your knowledge of quadratic functions and began to work with complex numbers. You learned how to graph the reciprocal function $f(x) = \frac{1}{x}$ and how to use the graph of a function to sketch its reciprocal. You have also added, subtracted, multiplied, and divided rational expressions.

Chapter 4 Review Test

Quadratic Functions and Rational Expressions

1. (a) If $z = -3 + 2i$ and $w = 2 - i$, then find $z + w$, $z - w$, $z\bar{z}$, and $\frac{z}{2w}$.

(b) Find $f(2 - 5i)$, where $f(x) = -2x^2 - 3x + 1$.

2. Simplify each of the following.

(a) $\frac{28x^3y}{-21xy^2}$

(b) $\frac{7}{x^2 - 4} - \frac{3}{x^2 + 5x + 6}$

(c) $\frac{a^2 - b^2}{a} \times \frac{3ab}{a^2 + 4ab + 3b^2} \div \frac{a^2 - ab}{a + 3b}$

(d) $(x - y)^3 + (x + y)^3 + 3(x + y)(x - y)^2 + 3(x - y)(x + y)^2$

3. (a) Find the coordinates of the vertex. State whether the function has a maximum or minimum value.

i. $f(x) = -3x^2 + 8x$

ii. $g(x) = (x - 1)(x + 5)$

iii. $h(t) = -4.9t^2 + 24.5t + 6$

(b) A car rental agency has 150 cars. The owner finds that, at a price of \$48 per day, he can rent all the cars. For each \$2 increase in price, the demand is less and 4 fewer cars are rented. For each car that is rented, there are routine maintenance costs of \$5 per day. What rental charge will maximize profit?

4. **Application:** Juliet has dropped her locket from her balcony so that Romeo, standing below, can put a note into it. He must then throw the locket back up to Juliet. The height of an object thrown vertically up is modelled by the quadratic function $s(t) = s_0 + v_0t - \frac{1}{2}gt^2$, where s_0 is the initial height above ground, v_0 is the initial velocity, g is the acceleration due to gravity, t is the time, and $s(t)$ is the height above the ground at time t. The balcony rail is 9 m above the ground, and the acceleration due to gravity is 9.8 m/s^2. Assume that the time it takes for the locket to fall 9 m is the same as the time it takes for Romeo to throw it back up to Juliet. What initial velocity must he give the locket if it is to reach Juliet? Show calculations or graphs, or both, to justify your answer.

5. **Knowledge and Understanding**

(a) For what values of k does the equation $kx^2 + k = 8x - 2kx$ have

i. two distinct real roots?

ii. one real root?

iii. no real roots?

(b) One root of a quadratic equation is $1 - 2i$. State the other root and write the equation in the form $ax^2 + bx + c = 0$.

6. The route for a fundraising marathon is 16 km long. Participants may walk, jog, run, or cycle. Make a table to show the estimated times for completing the marathon at the different speeds. Write an equation to describe the relation between speed and time and sketch its graph. What is the name of this relation?

7. **Communication:** Sketch the graph of $y = 3x - 4$ and use it to sketch the graph of $y = \frac{1}{3x - 4}$ on the same axes. State the domain and the range for the reciprocal function. Explain the properties of functions and their reciprocals that helped you sketch the second graph.

8. A formula for calculating the drug dose for young children is $\frac{(t^3 - 4t^2 - 5t)a}{22t^2 - 110t}$, where t is the age of the child in years and a is the adult dose. If the adult dose for a certain antibiotic is 200 mg, what dose does the formula indicate for a
   (a) 1-year-old child?
   (b) 5-year-old child?

   Answer to one decimal place.

9. Marissa and Jovanna enter a 200-km bicycle race. Marissa cycles 5 km/h faster than Jovanna, but her bicycle gets a flat tire, which takes one-half hour to repair. If the two girls finish the race in a tie, then how fast was each girl cycling? Answer to one decimal place.

10. **Thinking, Inquiry, Problem Solving**
    Prove that the triangle whose sides are $n^2 + 1$, $n^2 - 1$, and $2n$ is a right triangle.